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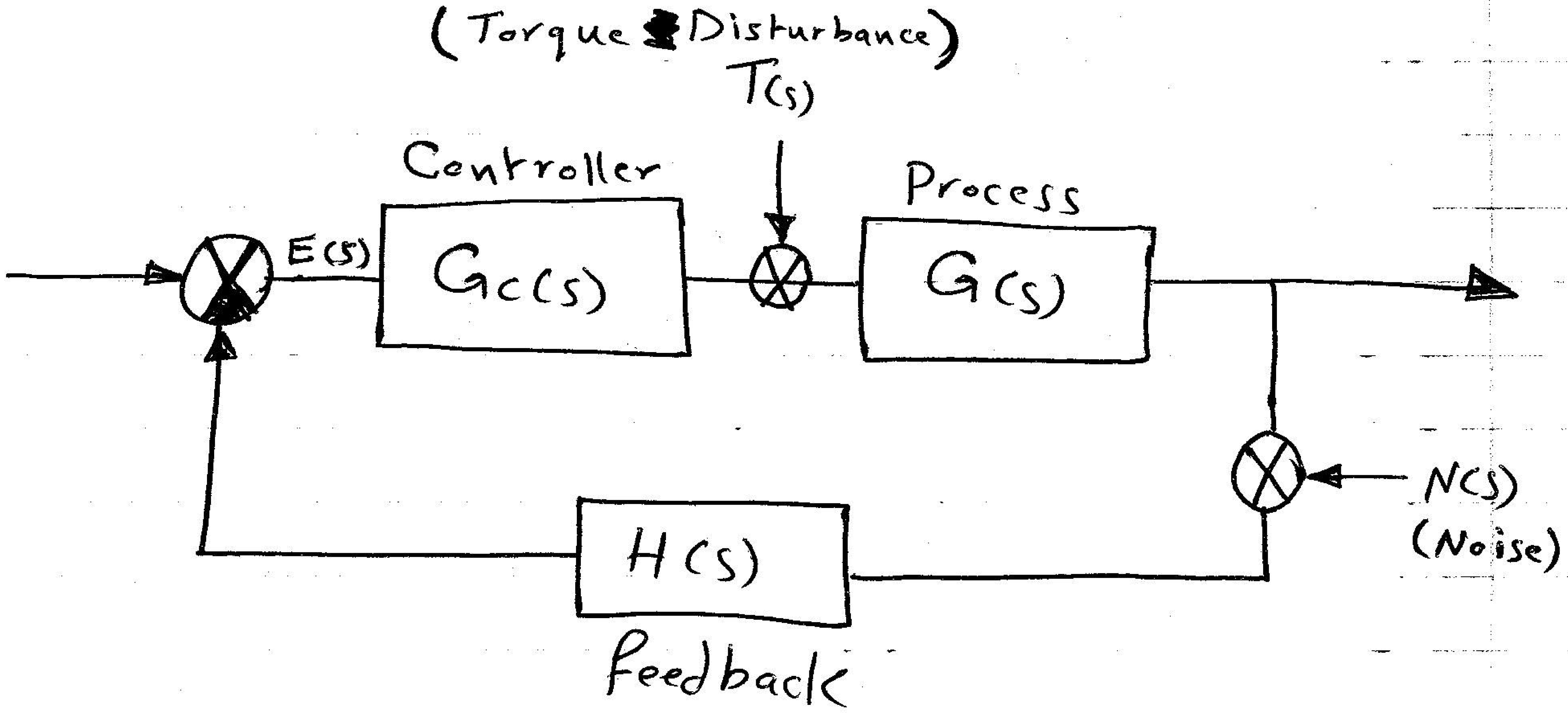
[Dorf 5.6]

The Steady State Error of a feedback Control System



$$E(s) = R(s) - C(s)\,H(s) \Longrightarrow H(s) = 1$$

$$\boxed{\therefore\ E(s) = R(s) - C(s)}$$

⊛ $E(s) = R(s) - C(s)$ $\quad H(s) = 1$

$$= R(s)\left[1 - \frac{C(s)H(s)}{R(s)}\right]$$

$$= R(s)\left[1 - \frac{G_c(s)\,G(s)\,H(s)}{1 + G_c(s)\,G(s)\,H(s)}\right]$$

$$= R(s)\left[\frac{1 + G_c(s)\,G(s)\,H(s) - G_c(s)\,G(s)\,H(s)}{1 + G_c(s)\,G(s)\,H(s)}\right]$$

$$= \boxed{\frac{1}{1 + G_c(s)\,G(s)\,H(s)} * R(s)}$$

* $\boxed{\text{If } R(s) = \frac{A}{s}}$ (Step Input).

$$E(s) = \frac{A/s}{1 + G_c(s)\, G(s)\, H(s)}$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s\, E(s)$$

$$= \lim_{s \to 0} s * \frac{A/s}{1 + G_c(s)\, G(s)\, H(s)}$$

$$= \frac{A}{1 + G_c(s)\, G(s)\, H(s)}$$

* The Loop transfer function is given by :-

$$\boxed{G_c(s)\, G(s)\, H(s) = \frac{K \prod_{i=1}^{M} (s - z_i)}{s^N \prod_{k=1}^{Q} (s - P_k)}}$$

~~[illegible]~~ (z_i : أصفار البسط) (P_k : أصفار المقام)

Ex

$$\frac{2}{3s^2} \times \frac{3(s+1)}{s^2+1} \; \frac{(s-2)}{s^3+s+1} = \frac{6(s+1)(s-2)}{3s^2(s^2+1)(s^3+s+1)}$$

K : حاصل قسمة ثابت الذي في البسط على الثابت الذي في المقام)

(total Gain ← 2 $= \frac{6}{3}$ = K في المثال السابق)

* The No of integration is often indicated by Labeling a system with a type Number that is equal to N.

* The steady State tracking error for a step input of magnitude "A" is given by :-

$$\boxed{e_{ss} = \frac{A}{1+K_p}}$$ for type Zero.

K_p : positioning Constant

* If the system has more than one integration, then e_{ss} = Zero

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$$E(s) = \frac{R(s)}{1+G_c(s)\,G(s)\,H(s)}$$

2

* $\boxed{\text{If } R(s) = \text{Ramp input} = \frac{A}{s^2}}$

$$e_{ss} = \lim_{s\to 0} s \frac{A/s^2}{1+G_c(s)\,G(s)\,H(s)} = \lim_{s\to 0} \frac{A}{s\,G_c(s)\,G(s)\,H(s)}$$

* If the system is type Zero $\Rightarrow$ ($N=0$)

$$\Rightarrow e_{ss} = \lim_{s\to 0} \frac{A}{s*K} = \frac{A}{0} = \boxed{\infty}$$

* If the system is type 1 $\Rightarrow N=1$

$$\Rightarrow e_{ss} = \lim_{s \to 0} \frac{A}{s * \frac{K_v}{s}} = \boxed{A/K_v}$$

K_v : Velocity Constant

$$G_c(s)\, G(s)\, H(s) = \frac{K \prod_{i=1}^{M} (s - z_i)}{s^N \prod_{k=1}^{Q} (s - P_k)}$$

* If the system is type 2 $\Rightarrow N=2$

$$\Rightarrow e_{ss} = \lim_{s \to 0} \frac{A}{s * \frac{k}{s^2}} = \frac{A}{\frac{k}{s}} = \frac{A}{\infty} = \boxed{\text{Zero}}$$

In the Case of Ramp Input, We need two Integrators to eliminate the error.

* $\boxed{3}$ $\boxed{\text{If } R(t) = \frac{At^2}{2} \Rightarrow R(s) = \frac{A}{s^3}}$

"Acceleration Input"

$$E(s) = \frac{R(s)}{1 + G_c(s) G(s) H(s)} = \frac{A/s^3}{1 + G_c(s) G(s) H(s)}$$

$$e_{ss} = \lim_{s \to 0} s \frac{A/s^3}{1 + G_c(s)\, G(s)\, H(s)} = \lim_{s \to 0} \frac{A}{s^2 G_c(s) G(s) H(s)}$$

a) $N=0 \Rightarrow e_{ss}=\infty$

$N=1 \Rightarrow e_{ss}=\infty$

$N=2 \Rightarrow e_{ss}=\dfrac{A}{K_a}$

$N=0 \Rightarrow G_c G H(s) = K_1$

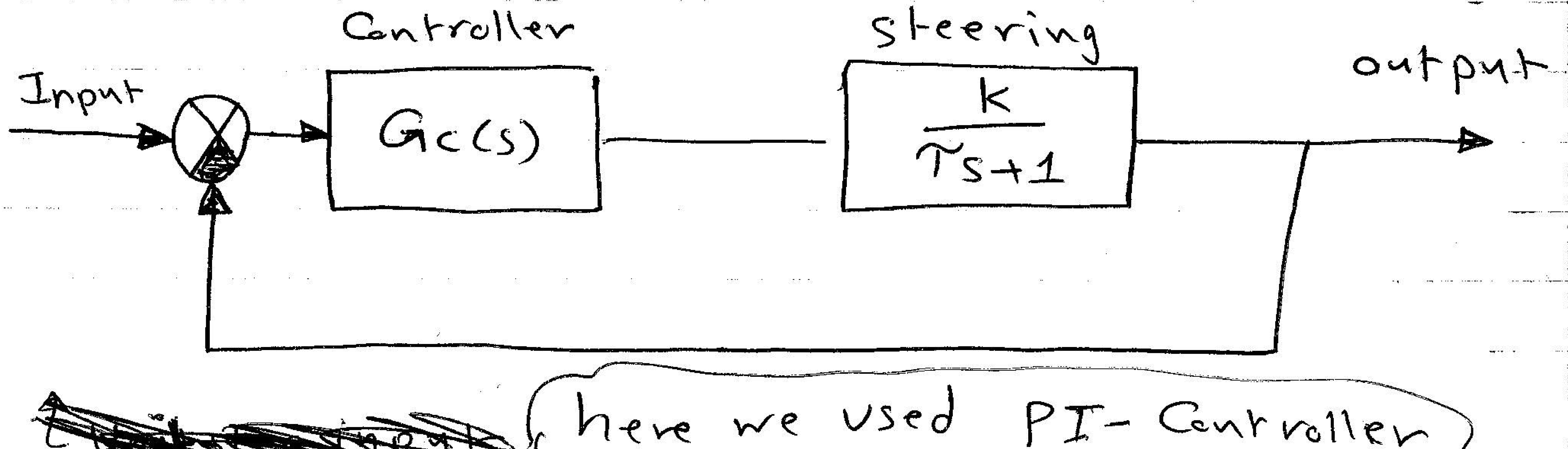
$N=1 \Rightarrow G_c G H(s) = \dfrac{K_2}{s}$

$N=2 \Rightarrow G_c G H(s) = \dfrac{K_3}{s^2}$

K_a = Acceleration Constant

Type \ Input	Unit step $r(t)=A$, $R(s)=A/s$	Ramp Input $r(t)=At$, $R(s)=A/s^2$	Acceleration Input $r(t)=At^2/2$, $R(s)=A/s^3$
$N=0$	$\dfrac{A}{1+K_p}$	∞	∞
$N=1$	Zero	$\dfrac{A}{K_v}$	∞
$N=2$	Zero	Zero	$\dfrac{A}{K_a}$

Ex: Mobile Robot may be designed as an assisting device or servant for disabled person.
- The steering Control system for a such robot Can be represented by



here we used PI- Controller "given in the question"

Case 1 : If $R(s) =$ ~~Ramp Input~~ Unit Step Input $= \frac{A}{s}$

If $G_c(s) = \frac{k_1 \cancel{s} + k_2}{\cancel{s}} = \boxed{k_1 + \frac{k_2}{s}}$

P I Controller

$$e_{ss} = \lim_{s \to 0} \cancel{s} \frac{A/\cancel{s}}{1 + G_c G H(s)} = \frac{A}{1 + G_c G H(s)}$$

$$G_c G H(s) = \frac{k_1 s + k_2}{\cancel{s}} * \frac{k}{Ts + 1} = \frac{k_2 * k}{\cancel{s}}$$

$$\therefore e_{ss} = \lim_{s \to 0} \cancel{s} \frac{A/s}{1 + G_c G H(s)} = \frac{A}{1 + G_c G H(s)} = \frac{A}{1 + \frac{k_2 k}{\cancel{s}}}$$

$$= \boxed{\text{Zero}}$$

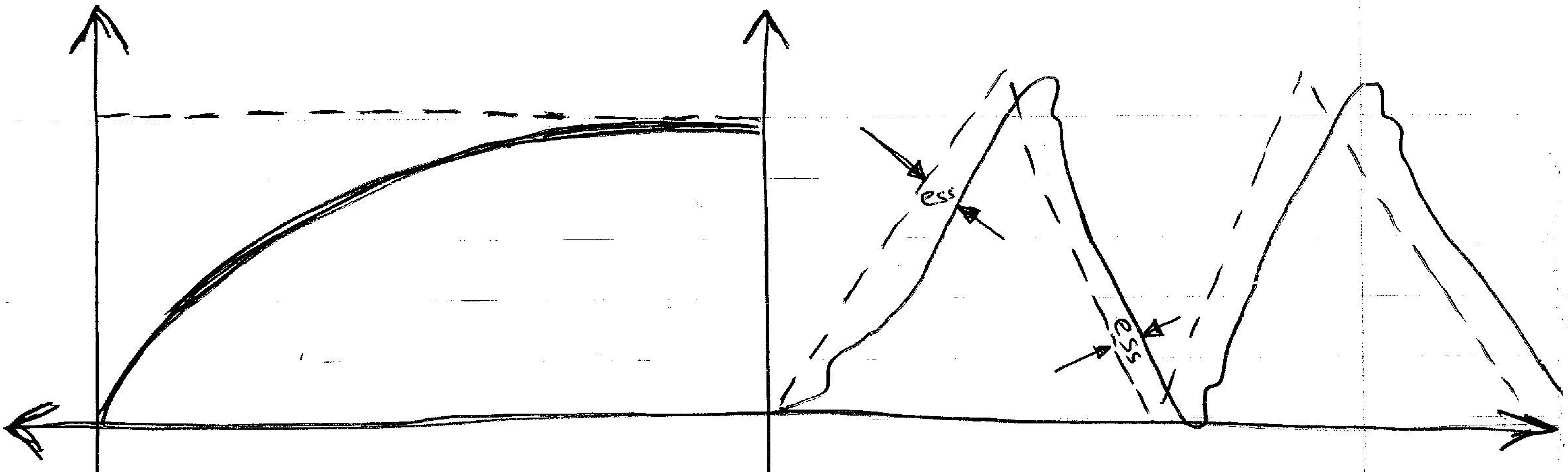
Case 2 : If $R(s) =$ Ramp Input $= \frac{A}{s^2}$ with PI Controller

$$e_{ss} = \lim_{s \to 0} s \frac{A/s^2}{1 + G_c G H(s)}$$

$$= \lim_{s \to 0} \frac{A}{s G_c G H(s)}$$

$$= \lim_{s \to 0} \frac{A}{s * \frac{k_2 k}{\cancel{s}}} = \boxed{\frac{A}{k_2 K}}$$

$K_2 K$: Velocity Constant



$R(s) = A/s$ $\qquad$ $R(s) = A/s^2$

In the case of Ramp Input, we need <u>two</u> Integrators to Eliminate the error.

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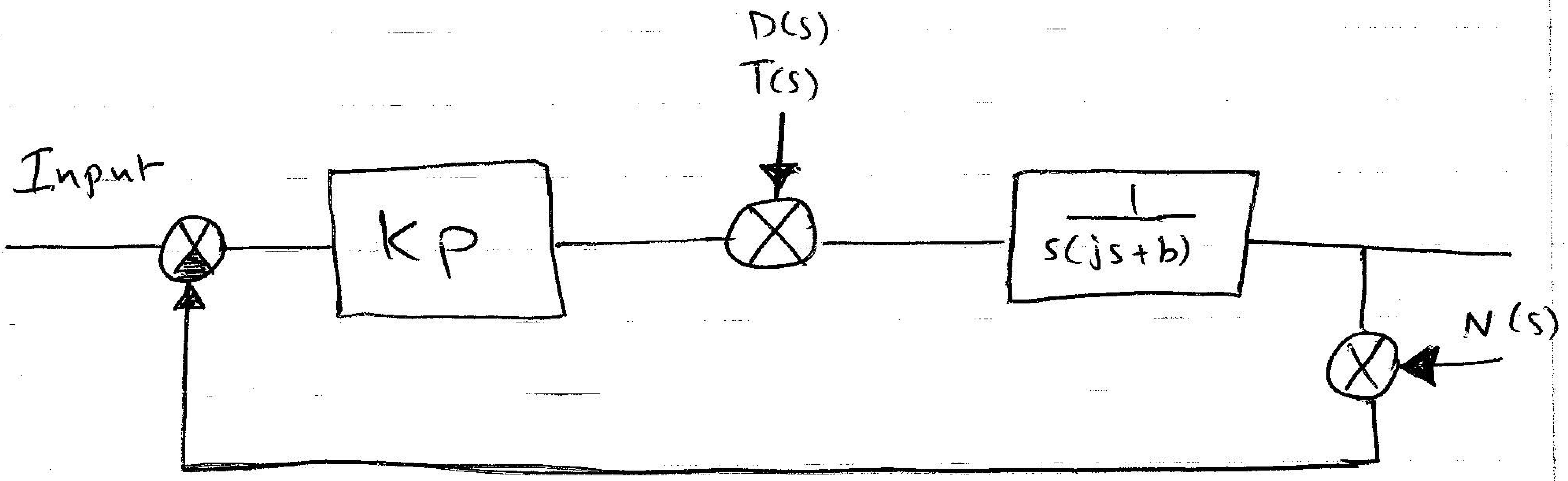
① Response to torque (Proportional Controller)



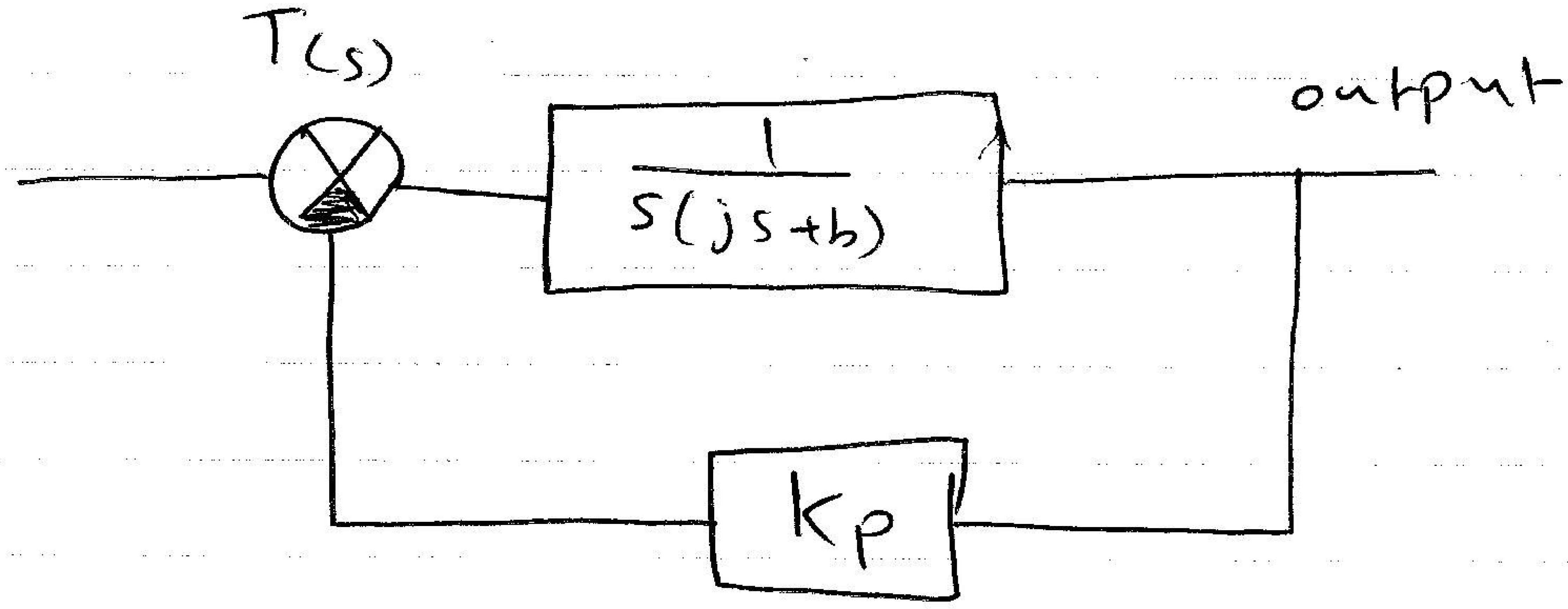
① Set Input to Zero

② Set N(s) = Zero

③ Rearrange the Control diagram

Input = 0
N(s) = 0

⇒ Now Rearrange the control diagram as follows :-



⊛ E(s) = Input − Output
= Zero − Output
= Zero − C(s) = $\boxed{-C(s)}$

⊛ $$\frac{C(s)}{D(s)} = \frac{\frac{1}{s(js+b)}}{\frac{1}{s(js+b)} * kp + 1} = \frac{1}{s(js+b)+kp}$$

$$= \boxed{\frac{1}{js^2+bs+kp}}$$

⊛ ∴ $$\frac{E(s)}{D(s)} = \frac{-C(s)}{D(s)} = \boxed{\frac{-1}{js^2+bs+kp}}$$

$$\boxed{\therefore\ E(s) = \frac{-Ds}{js^2+bs+kp}}$$

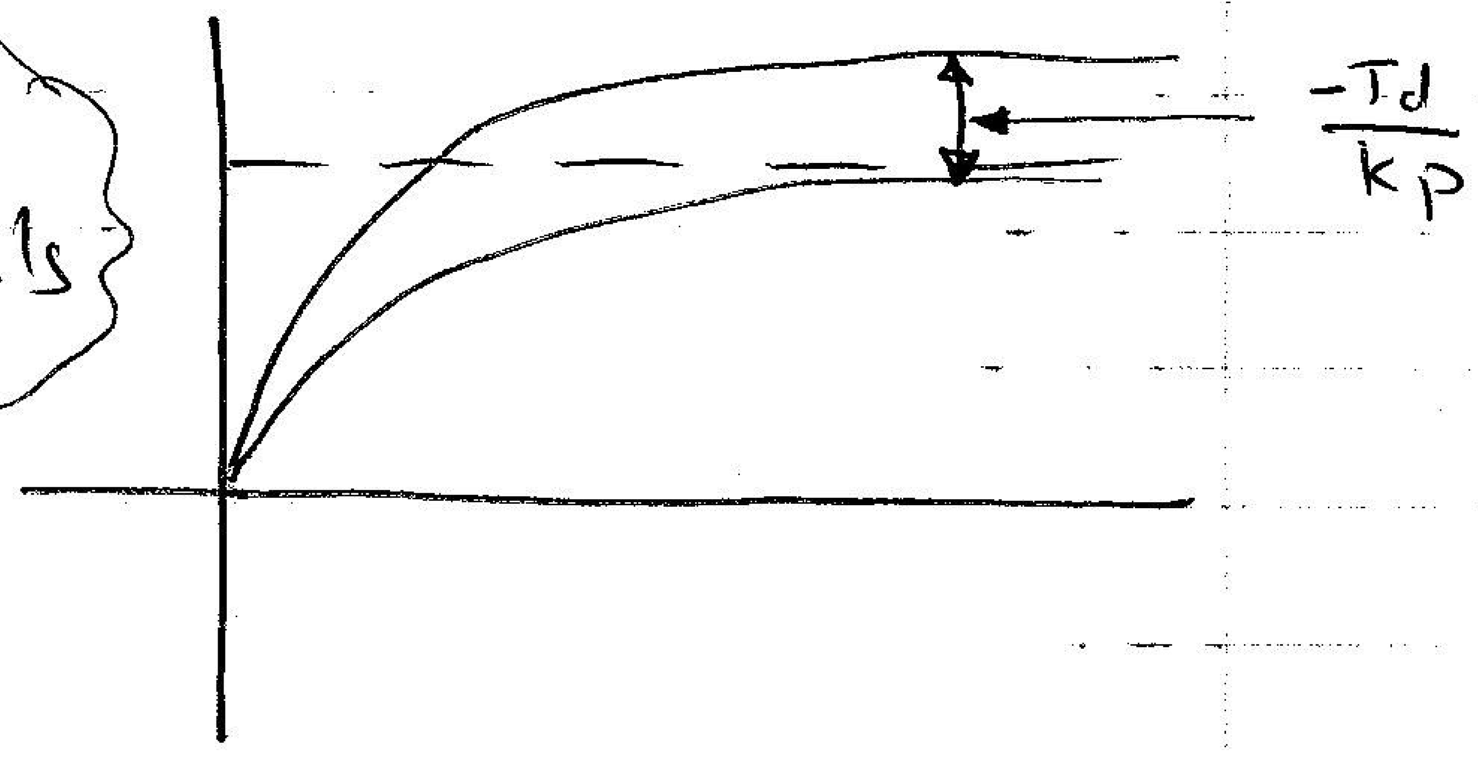
D(s) is a unit Step disturbance $\Rightarrow D(s) = \frac{Td}{s^1}$

$$\therefore e_{ss} = \lim_{s \to 0} s \, \frac{\frac{-Td}{s}}{js^2 + bs + kp}$$

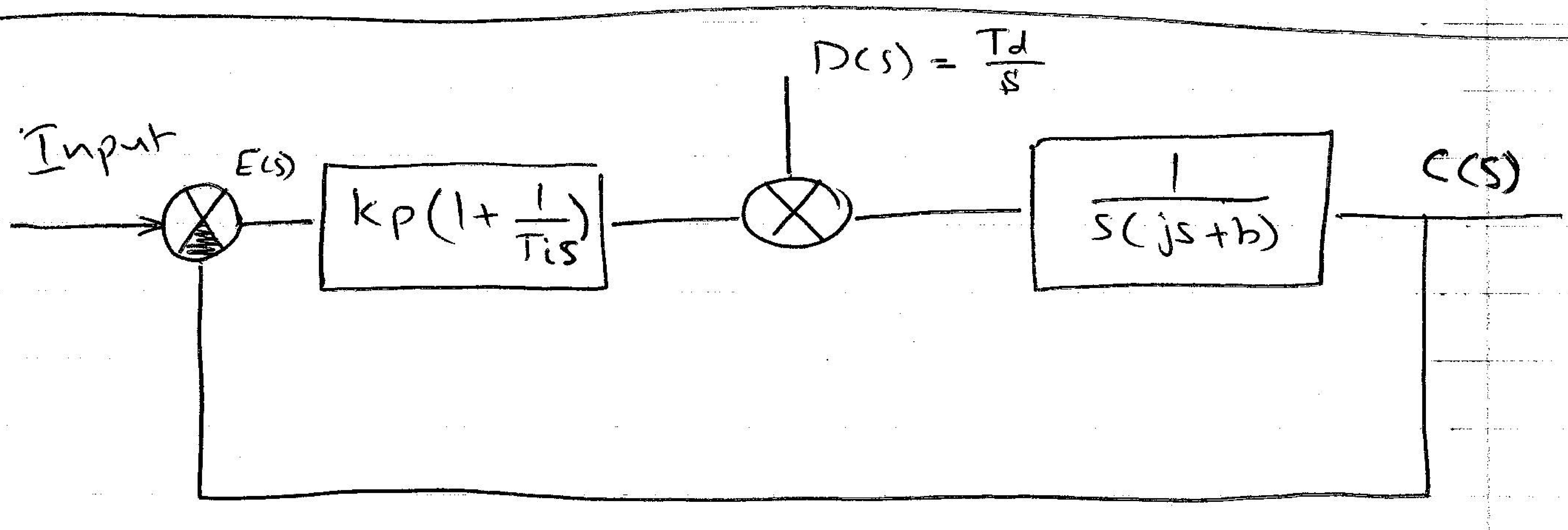
$$= \lim_{s \to 0} \frac{-Td}{js^2 + bs + kp} = \frac{-Td}{j(0)^2 + b(0) + kp}$$

$$= \boxed{\frac{-Td}{kp}}$$

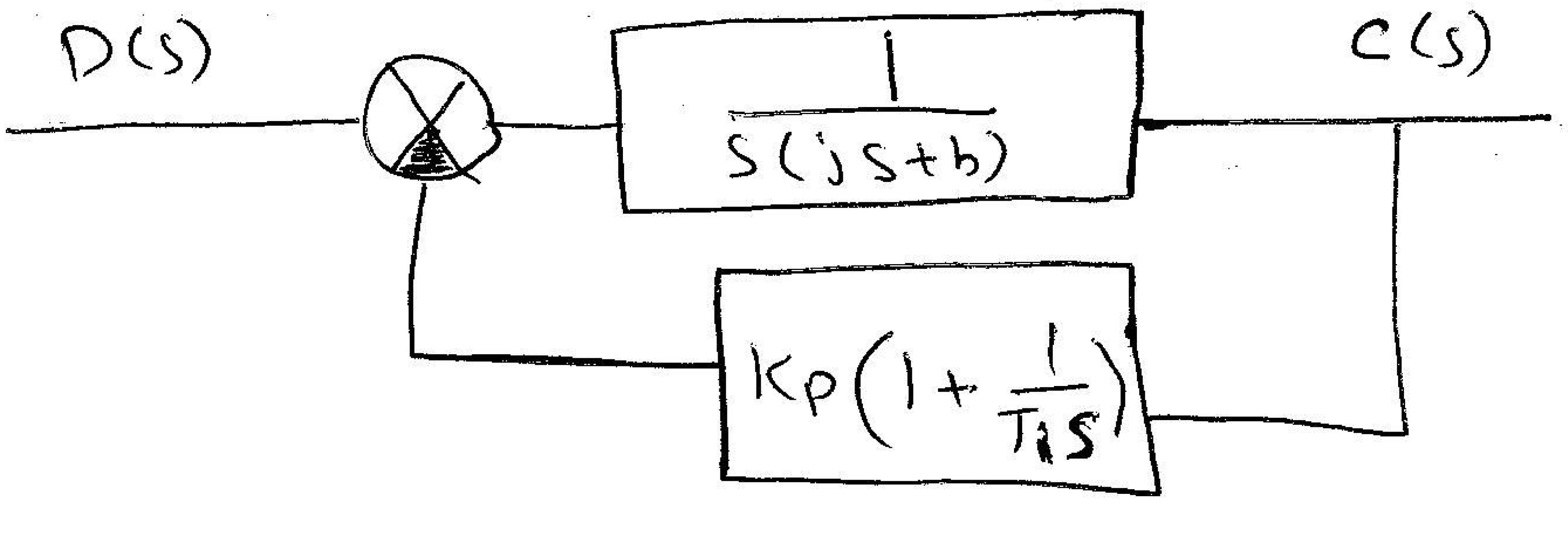
∴ The P-controller Cannot eliminate the error and equals it to zero, but it can decrease the error.



② Response to torque (PI-Controller)



$\Rightarrow$ Rearrange the Control diagram : (input = zero)



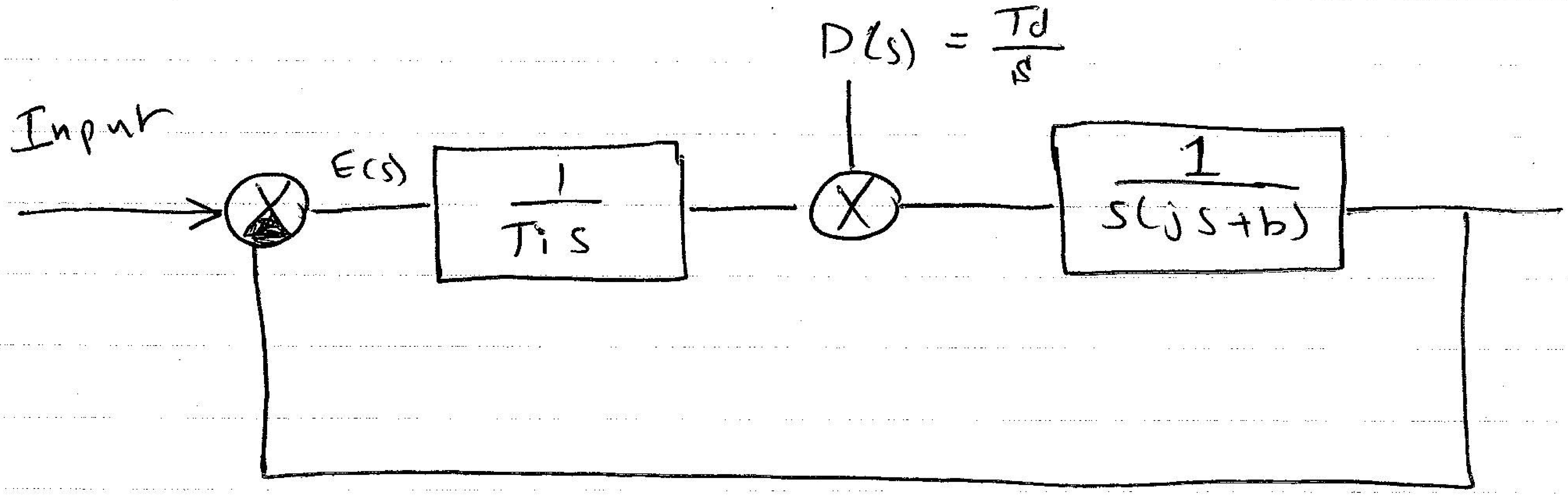
$$\therefore \frac{E(s)}{D(s)} = \frac{-s}{Js^3 + bs^2 + K_p s + \frac{K_p}{T_i}}$$

$$\Rightarrow e_{ss} = \lim_{s \to 0} s \frac{-s \, \frac{T_d}{s}}{Js^3 + bs^2 + K_p s + \frac{K_p}{T_i}}$$

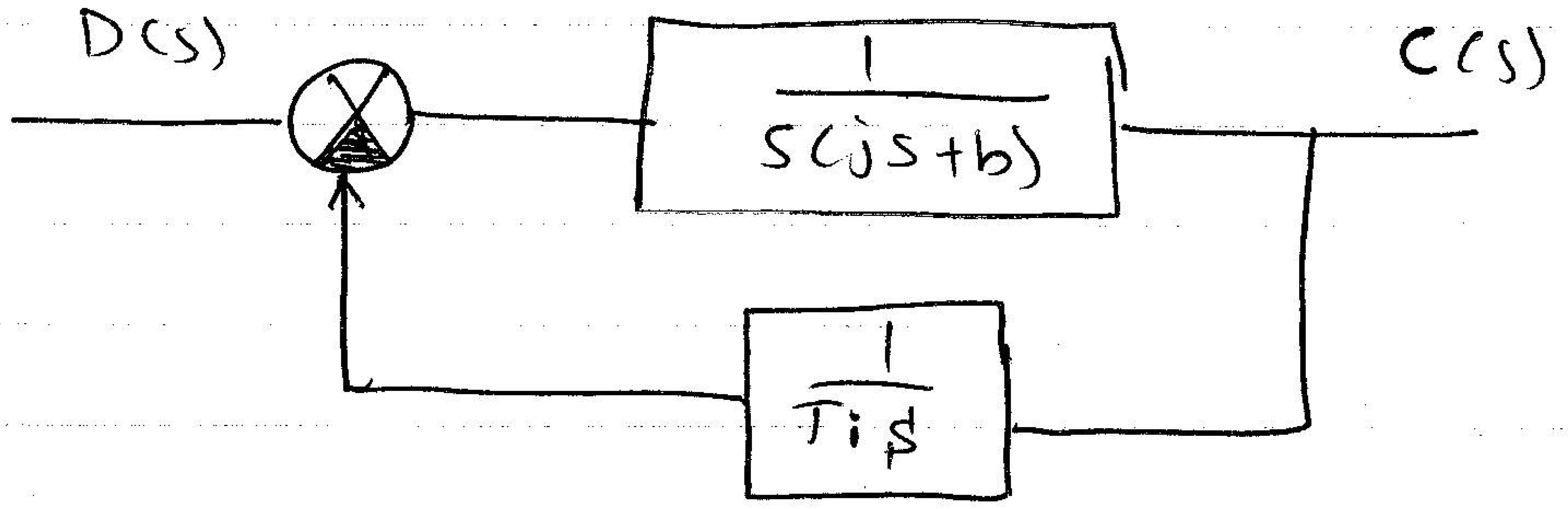
$$= \boxed{\text{Zero}}$$

∴ Using the PI-Controller eliminate the error

③ Response to torque (I-Controller only)



⇒ Rearrange the control diagram (Input = 0)



$$\frac{C(s)}{D(s)} = \frac{\frac{1}{s(js+b)}}{\frac{1}{s(js+b)} * \frac{1}{T_i s} + 1} = \frac{1}{\frac{1}{T_i s} + s(js+b)}$$

$$= \frac{T_i s}{1 + T_i s^2 (js+b)}$$

$$= \frac{T_i s}{1 + T_i j s^3 + T_i b s^2} = \frac{C(s)}{D(s)}$$

$$\therefore e_{ss} = \lim_{s \to 0} s \frac{T_i s * \frac{T_d}{s}}{1 + T_i j s^3 + T_i b s^2}$$

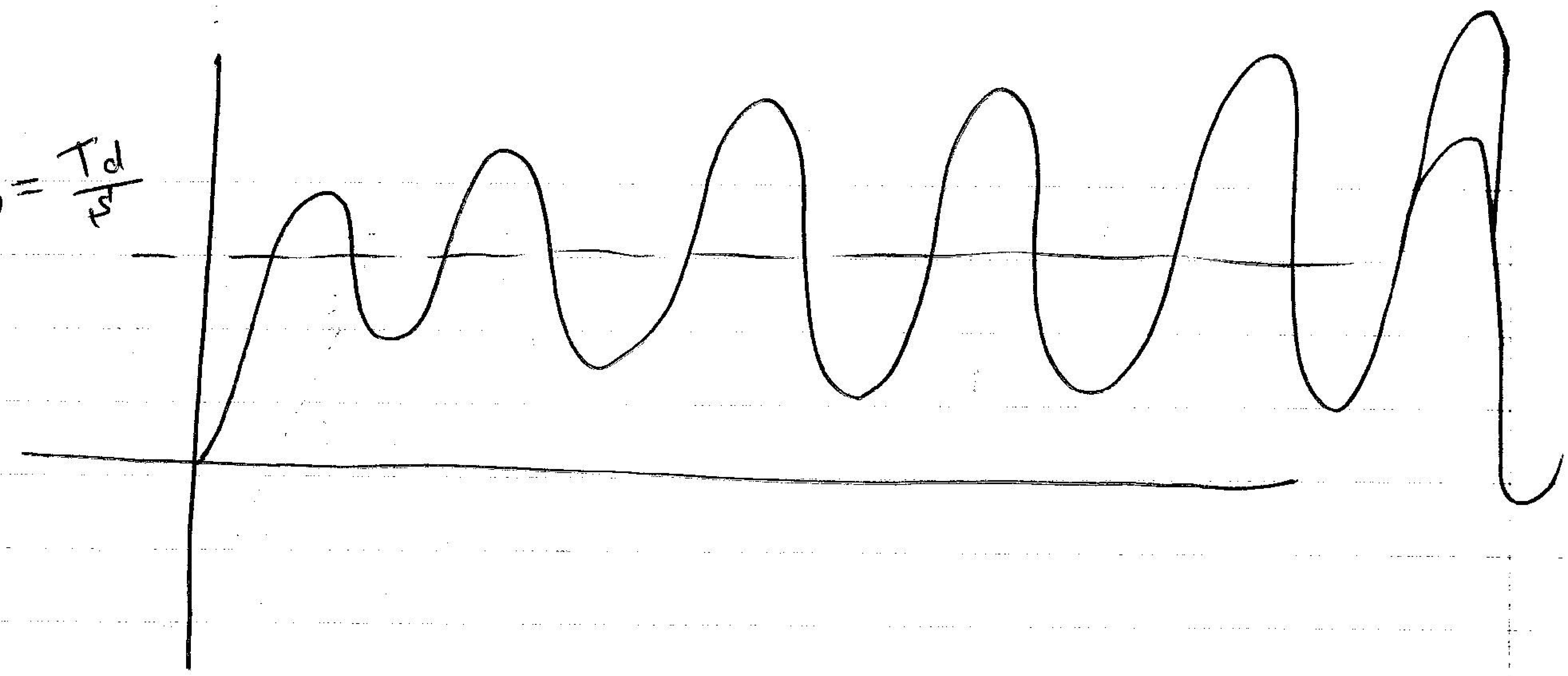
$$= \boxed{\text{Zero}}$$

$$\frac{C(s)}{D(s)} = \frac{T_i s}{T_i j s^3 + T_i b s^2 + 1}$$

s^3	$T_i j$	0
s^2	$T_i b$	1
s	$\frac{-j}{b}$	0
1	1	

The system is unstable !!

$D(s) = \frac{T_d}{s}$



⊛ :- We must Use PI- Controller to eliminate the error and to maintain the stability of the system.

⊛ Using I-Controller only eliminate the errors but the system is "Unstable".

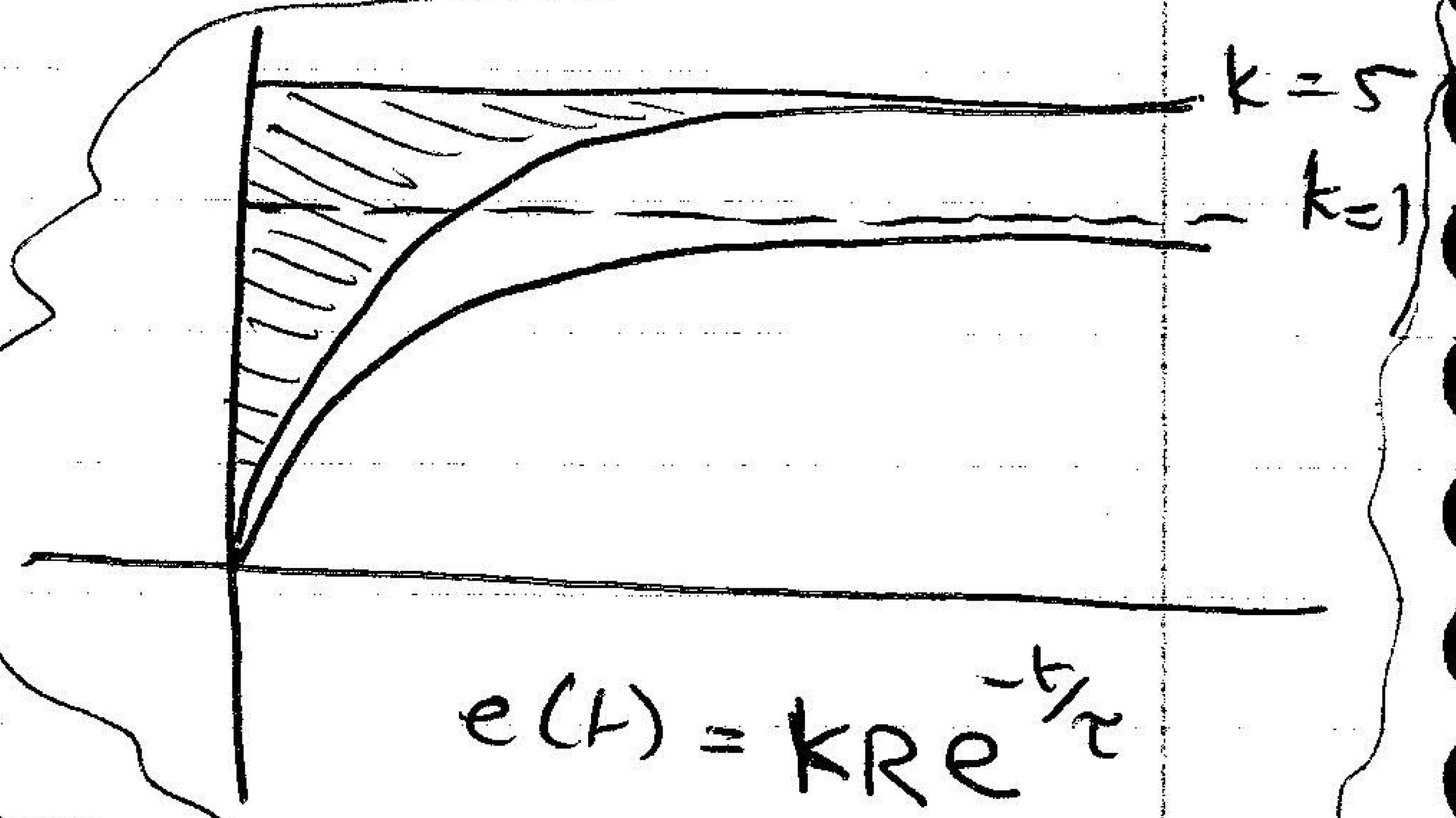
البحث في أدنى قيمة للخطأ

Performance Indices [Dorf 5.7]

⊛ A performance index : is a quantitive measure of the performance of a system, and is chosen so that emphasis is given to the important system specifications

⊛ a system is Considered an <u>optimum Control system</u> when the system parameters are adjusted so that the Index reach its extremum [minimum] Value.

⊛ a performance Index must be a number that is always <u>Positive</u>



$e(t) = kRe^{-t/\tau}$

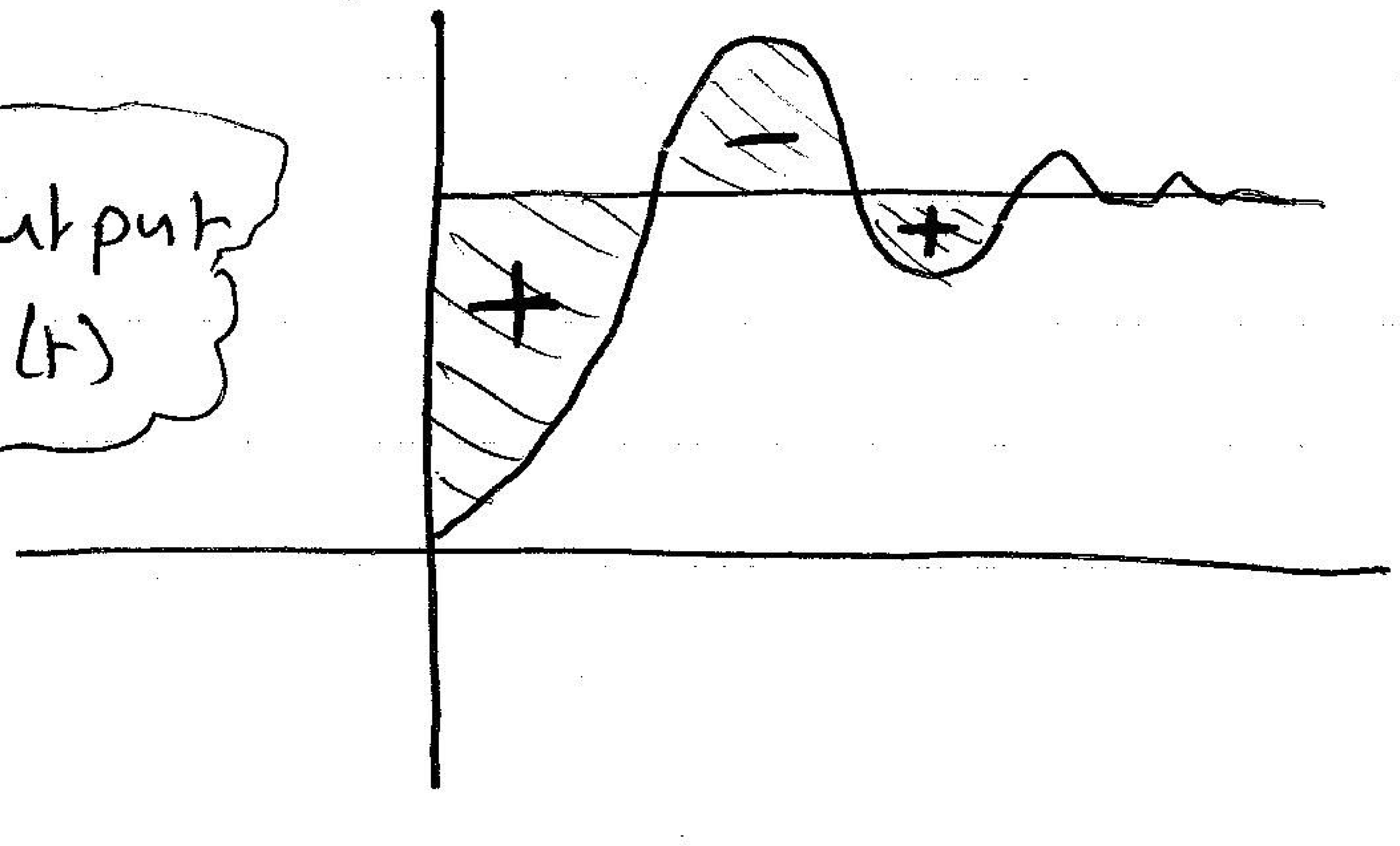
$e(\infty) = e_{ss} = \underline{kR}$

Performance Index Symbol : [I]

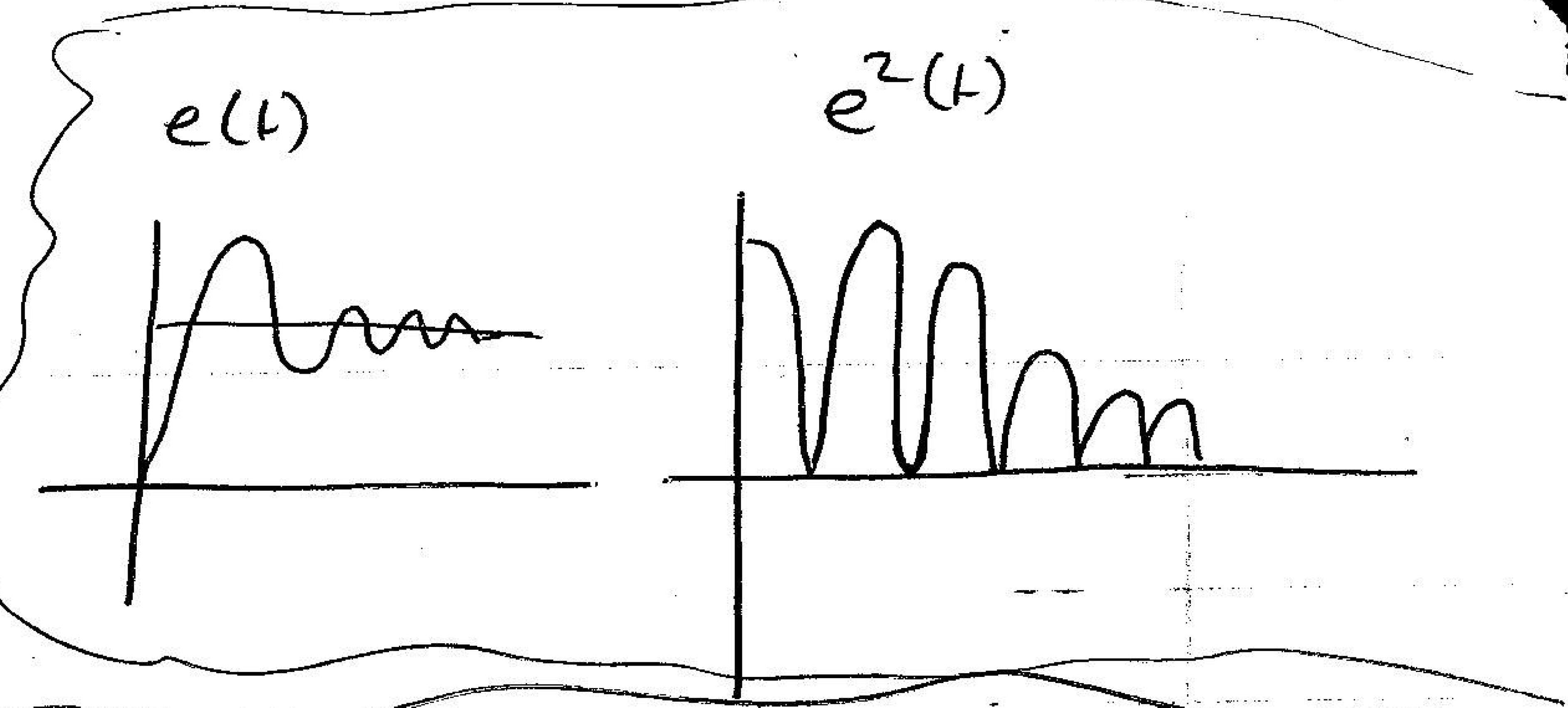
Error Criterion :-

[1.] Integral of the Square of the error (ISE)

error = Input − output
= $R(t) - C(t)$

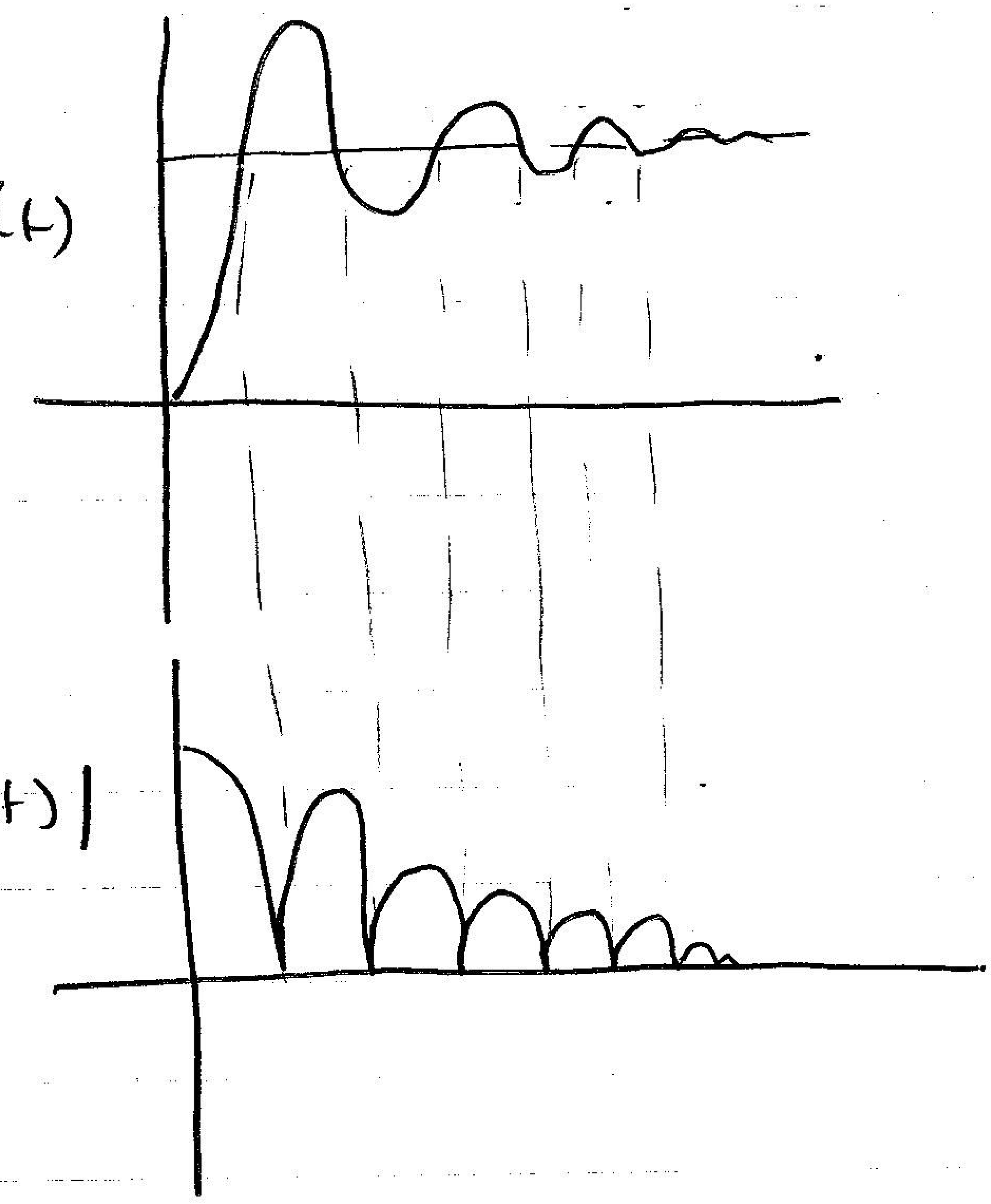


$$I = \int_0^{T_s \; (T_s \text{ or } \infty)} e^2(t) \cdot dt$$



2. Integral of the absolute value of the error (IAE)

$$I = \int_0^{T_s \; (T_s \text{ or } \infty)} |e(t)| \, dt$$



3. Integral of time multiplied by absolute error (ITAE)

$$I = \int_0^{T_s \; (T_s \text{ or } \infty)} t \, |e(t)| \, dt$$

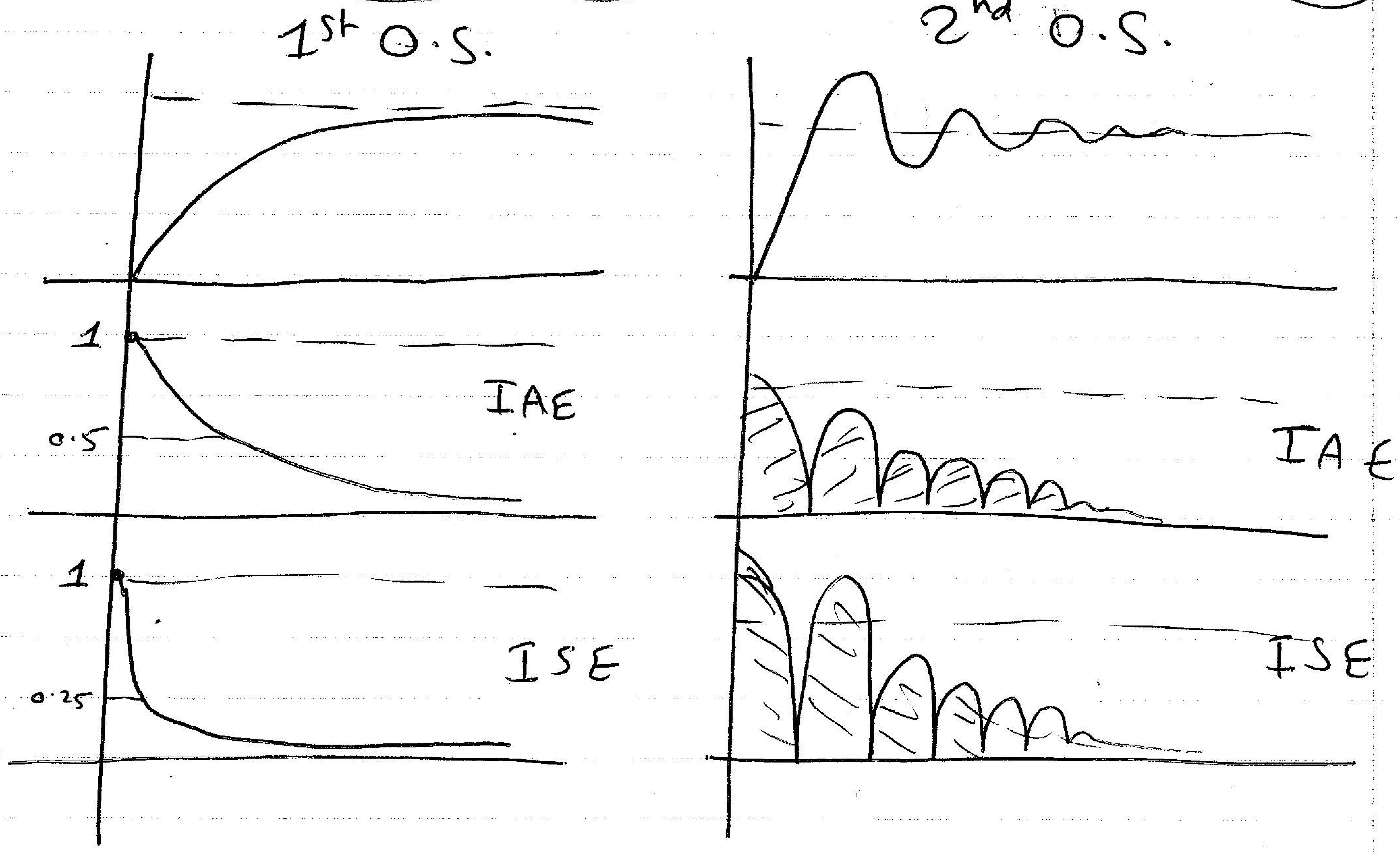
to reduce the Contribution of Initial error.

$t = \infty$

$e = 0$

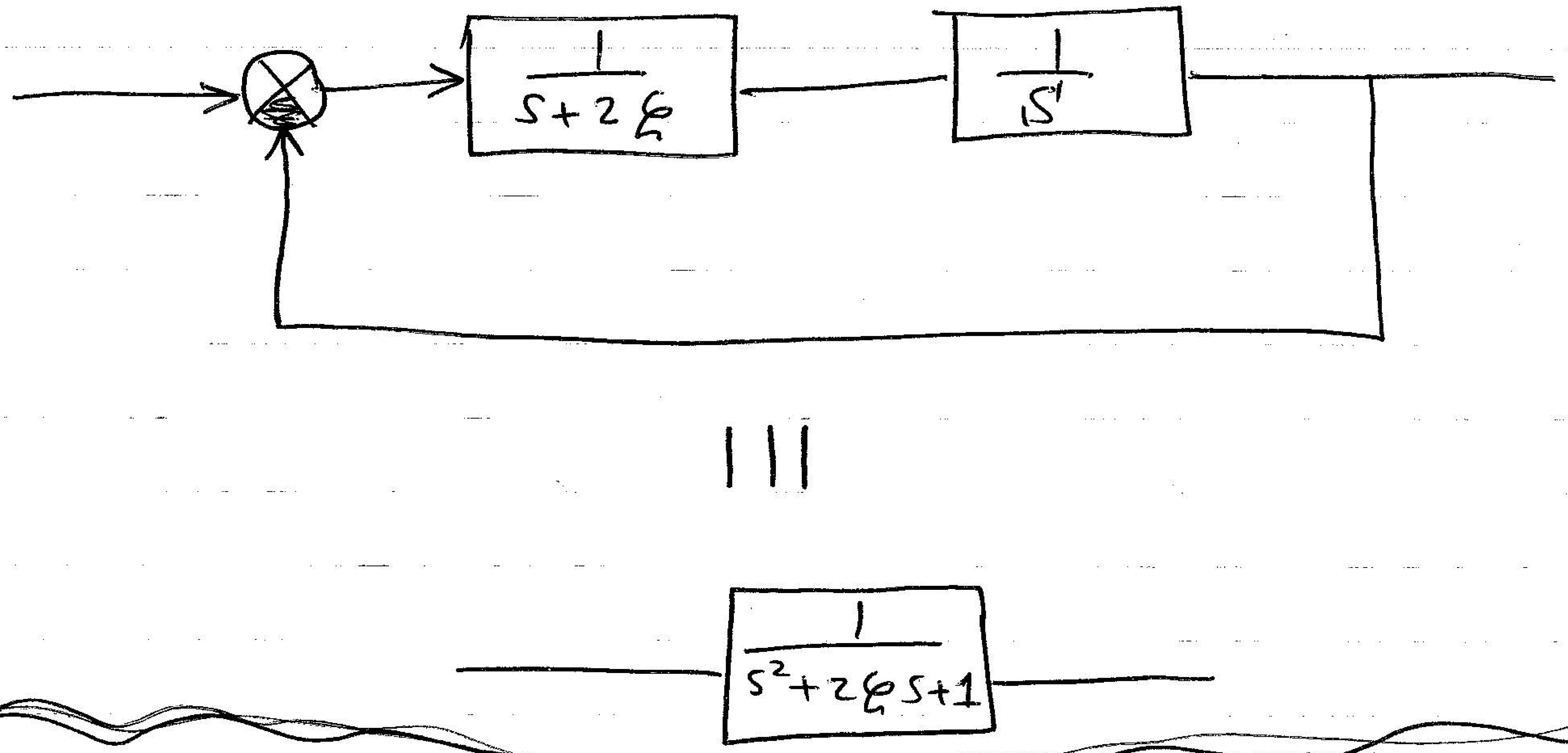
4. Integral of time multiplied by square error (ITSE)

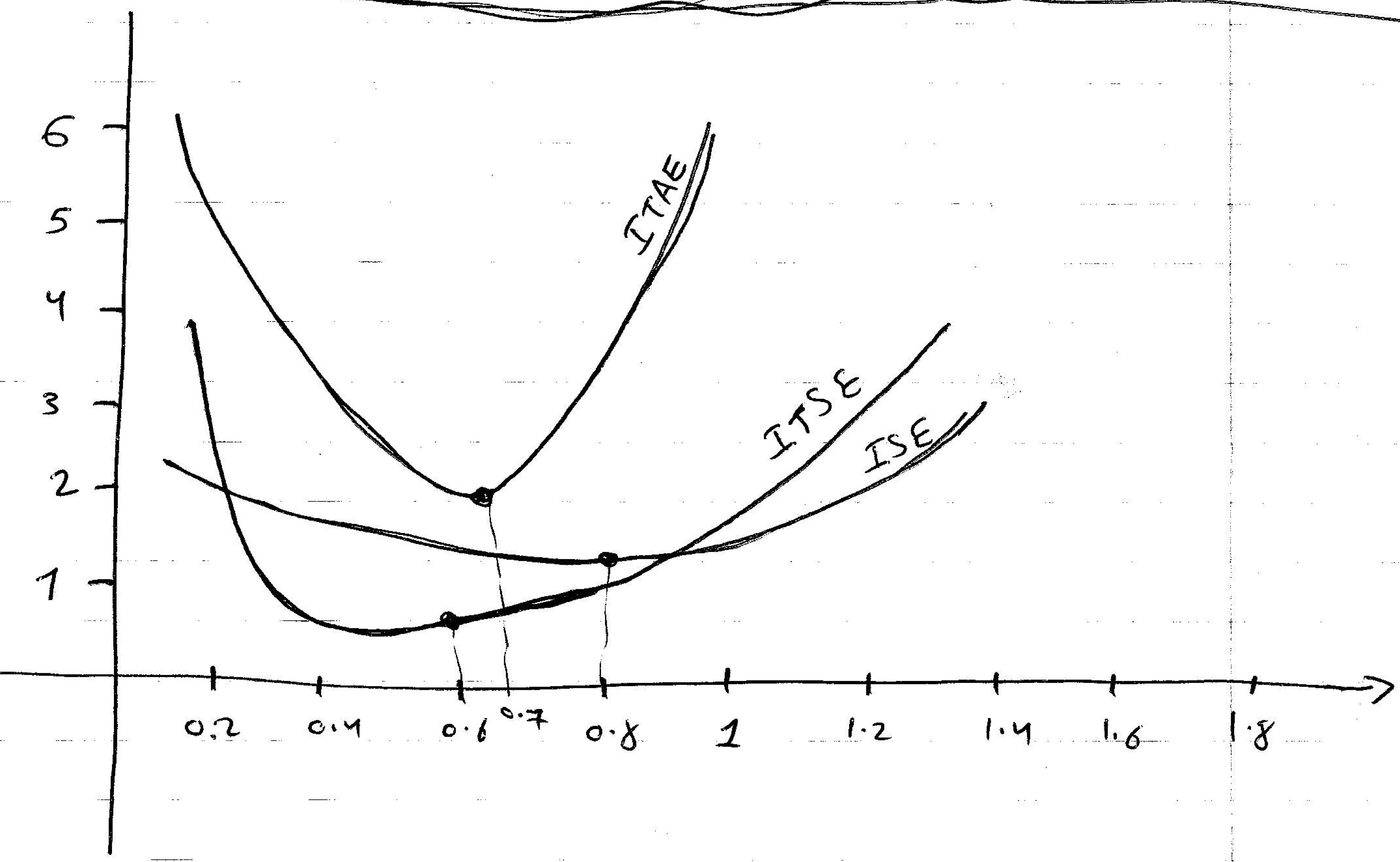
$$I = \int_0^{T_s} t\, e^2(t)\, dt$$



Example

a single loop feed back Control system is shown below where Wn is normalized ($W_n = 1$).
find the value of (ξ) so that the system is called optimum Controlled.



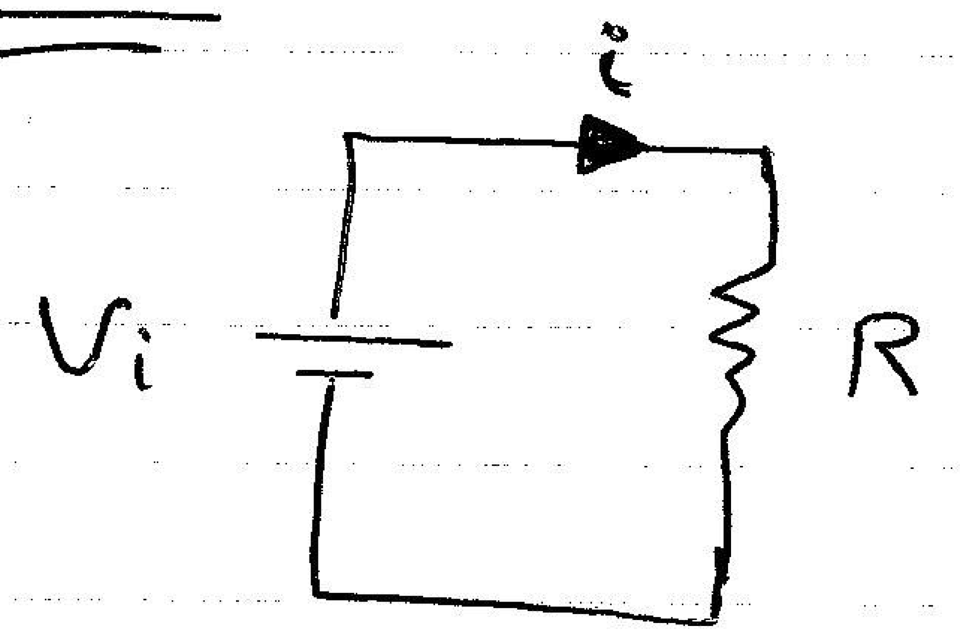


- optimal Value of ξ based on ITAE $= \boxed{0.7}$ ← ξ
- optimal Value of ξ based on ITSE $= \boxed{0.6}$
- optimal Valuof ξ based on ISE $= \boxed{0.8}$

Control

Linear — Non-linear

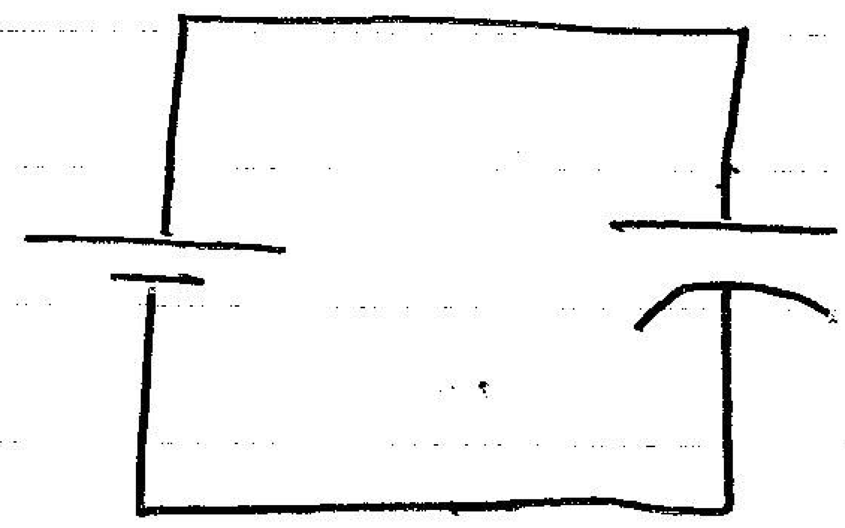
(Linear)



$V = IR$

$V(s) = I(s) * R$

(Linear)

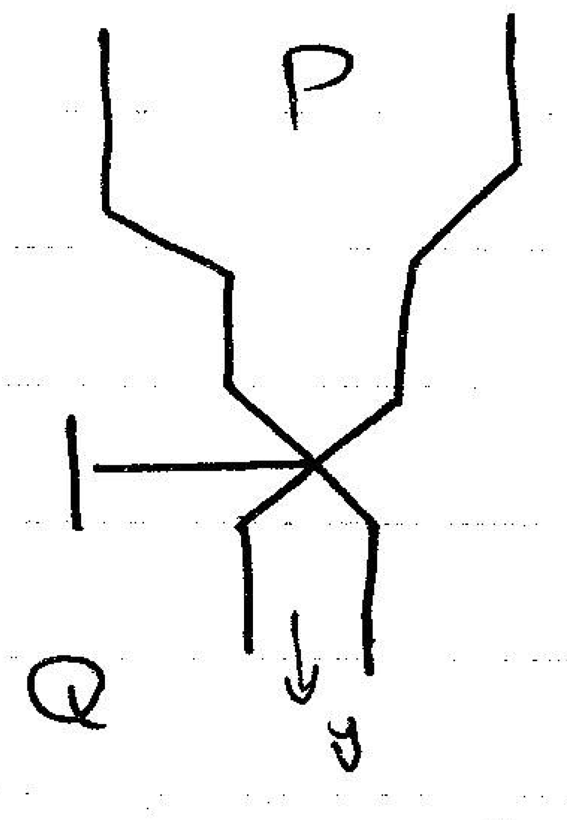


$V = \frac{1}{C} \int i \, dt$

$V(s) = \frac{1}{C} \frac{I(s)}{S}$

(Linear)

(Non-Linear)



$Q = \frac{P}{R}$ $\quad$ $P = \rho g h$

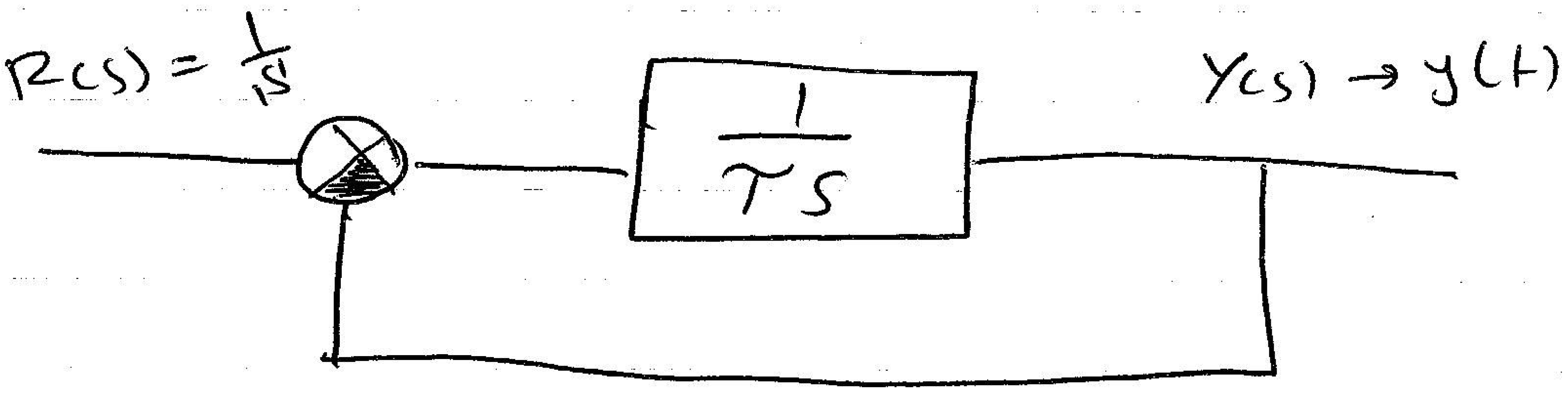
$Q \propto \sqrt{\Delta P}$

In Control 2, We Study linear Control Systems only!

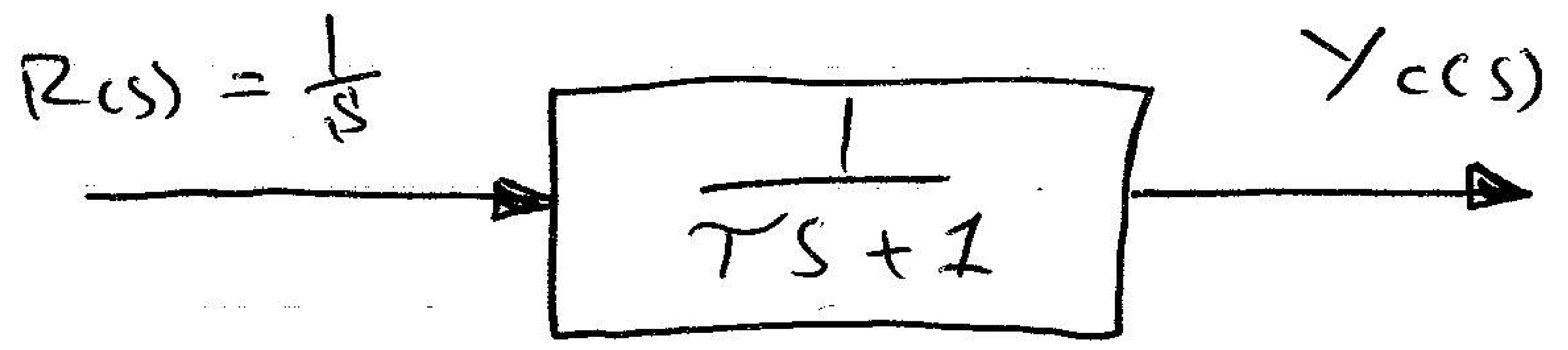
Continue ;

Ex For the following system, find the performance Index based on IAE , ISE , ITAE , ITSE .

Note : Performance Index based on time domain .



$\equiv$



$$y(t) = 1 - e^{-t/\tau}$$

$$e(t) = r(t) - y(t) = \boxed{e^{-t/\tau}}$$

---

1. $$I_{IAE} = \int_0^\infty |e^{-t/\tau}|\, dt = \int_0^\infty e^{-t/\tau}\, dt$$

$$= \frac{e^{-t/\tau}}{-\frac{1}{\tau}} = -\tau e^{-t/\tau} \Big]_0^\infty$$

$$= 0 - -\tau = \boxed{\tau}$$

---

2. $$I_{ISE} = \int_0^\infty (e^{-t/\tau})^2\, dt = \int_0^\infty e^{-2t/\tau}\, dt$$

$$= \int_0^\infty e^{-t/\left(\frac{\tau}{2}\right)} = \boxed{\frac{\tau}{2}}$$

3. $I_{ITAE} = \int_0^\infty t\left|e^{-t/\tau}\right| dt$

$$= \int_0^\infty t\, e^{-t/\tau}\, dt$$

$$= -\tau t e^{-t/\tau} - \int -\tau e^{-t/\tau} dt$$

$$= -\tau t e^{-t/\tau} + \tau \int e^{-t/\tau} dt$$

$$= -\tau t e^{-t/\tau} - \tau^2 e^{-t/\tau} \Big]_0^\infty$$

$$= [0-0] - [0 - \tau^2] = \boxed{\tau^2}$$

4. $I_{ITSE} = \int_0^\infty t\left(e^{-t/\tau}\right)^2 dt = \int_0^\infty t\, e^{-2t/\tau}\, dt$

Suppose $L = \tau/2$

$\therefore \int_0^\infty t\, e^{-t/L}$ (نفس حل القسم الثالث اعلاه ↑)

$\therefore \int_0^\infty t\, e^{-t/L} = L^2$

but $L = \tau/2$

$\therefore$ Answer $= \boxed{\frac{\tau^2}{4}}$

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Back to Sensitivity

There is a mathematical Method to compute $\sqrt{X}$.

Ex We need to find $\sqrt{5}$

Solution

$y_o = \sqrt{x_o}$

$2 = \sqrt{4}$

$y_o + \Delta y = \sqrt{5}$

$2 + \Delta y = \sqrt{5}$

$dx = 1 \approx \Delta X$

$$y = \sqrt{x}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\frac{\Delta Y}{\Delta X} = \frac{1}{2\sqrt{x_o}}$$

$$\Delta Y = \frac{1}{2\sqrt{x_o}} * \Delta X = \frac{1}{2\sqrt{4}} * 1 = \frac{1}{4} * 1 = \frac{1}{4}$$

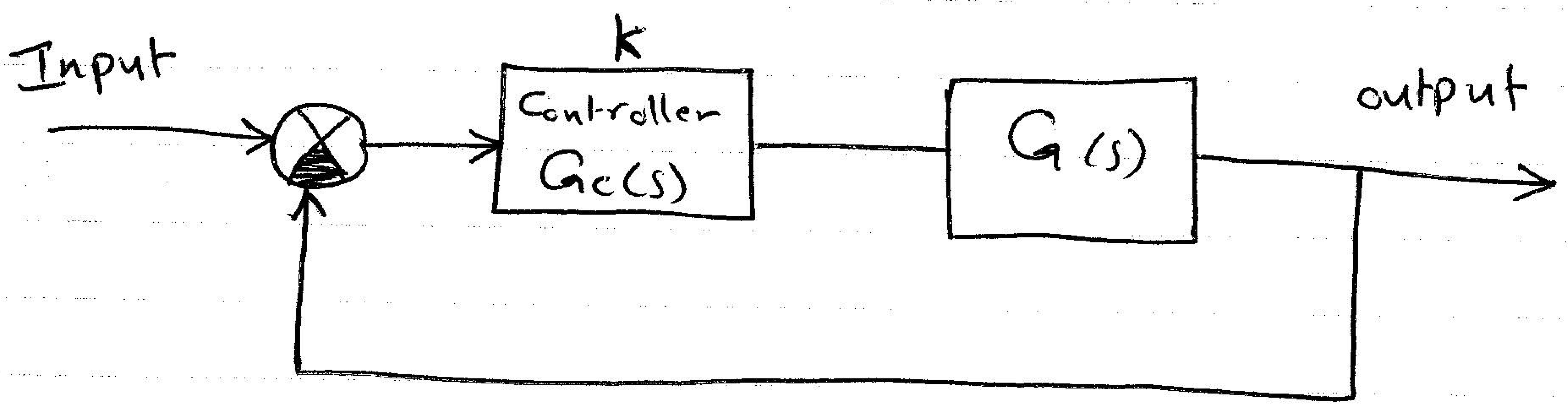
$$= \boxed{0.25}$$

$$\therefore \sqrt{5} = Y + \Delta Y = 2 + 0.25 = \boxed{2.25}$$

System Sensitivity : is the ratio of change in the System transfer function to the change of a process transfer function (or parameter) for a small ~~incremental change~~. incremental change.

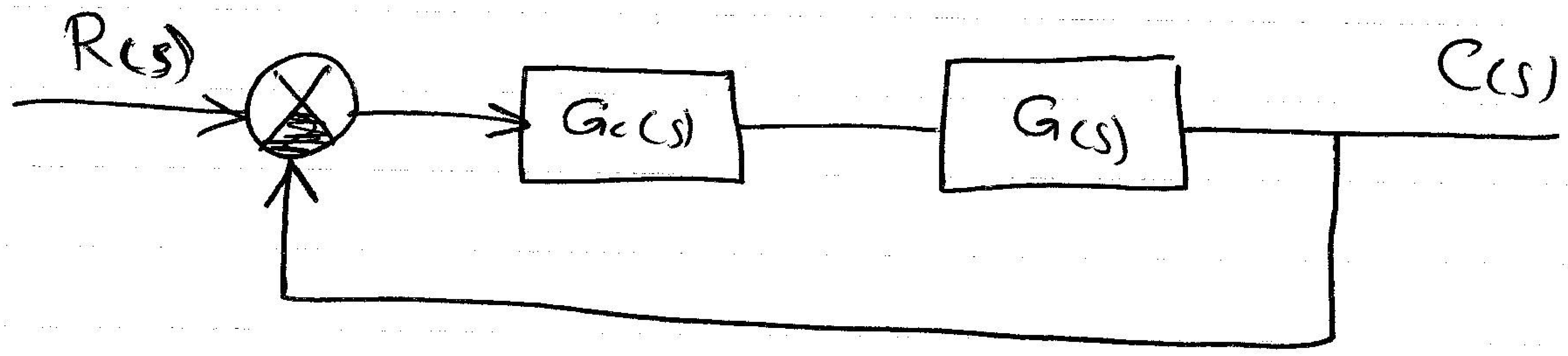
$$T(s) = \frac{Y(s)}{R(s)}$$

$$\boxed{S_G^T = \frac{dT/T}{dG/G} = \frac{d\ln T}{d\ln G}}$$



$$\boxed{\therefore \quad S_G^T = \frac{dT/T}{dG/G} = \frac{d\ln T}{d\ln G} = \frac{dT}{dG} \cdot \frac{G}{T}}$$

<u>Ex</u> for the following system, find the sensitivity of the system to process.



$$T(s) = \frac{G_c G}{1 + G_c G}$$

$\Rightarrow$

$$S_G^T = \frac{dT}{dG} * \frac{G}{T}$$

$$\Rightarrow \frac{dT}{dG} = \frac{(1+G_c G)\, G_c - G_c\, G\, G_c}{(1+G_c G)^2}$$

$$= \frac{G_c + \cancel{G_c^2 G} - \cancel{G_c^2 G}}{(1+G_c G)^2} = \boxed{\frac{G_c}{(1+G_c G)^2}}$$

$$S_G^T = \frac{dT}{dG} * \frac{G}{T} = \frac{\cancel{G_c}}{(1+G_c G)^2} * \frac{\cancel{G}}{\frac{\cancel{G_c G}}{1+G_c G}}$$

$$= \frac{1+G_c G}{(1+G_c G)^2} = \boxed{\frac{1}{1+G_c G}}$$

---

* for the upper system, find the sensitivity of the system to feedback.

$$S_H^T = \frac{dT}{dH} * \frac{H}{T}$$

$$T(s) = \frac{G_c G}{1 + G_c G H}$$

$$\frac{dT}{dH} = \frac{(1+G_cGH)(0) - (G_cG)(G_cG)}{(1+G_cGH)^2}$$

$$= \frac{-G_c^2 G^2}{(1+G_cGH)^2}$$

$$\therefore S_H^T = \frac{dT}{dH} * \frac{H}{T} = \frac{-G_c^{\cancel{2}} G^{\cancel{2}}}{(1+G_cGH)^{\cancel{2}}} * \frac{H}{\frac{\cancel{G_cG}}{\cancel{1+G_cGH}}}$$

$$= \boxed{\frac{-G_cGH}{1+G_cGH}}$$

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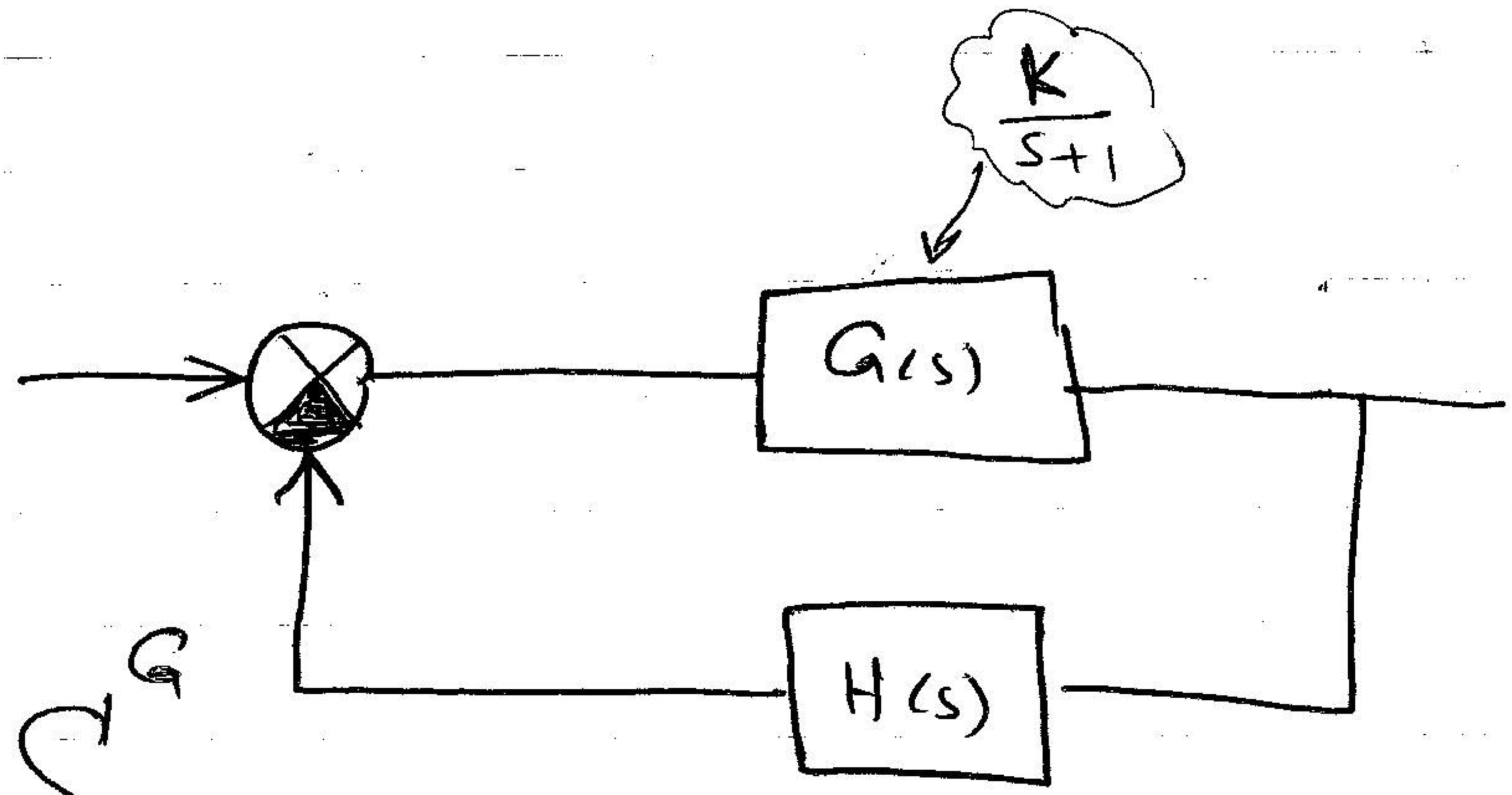
Chain Rule

$$S_{\alpha}^{T} = S_{G}^{T} * S_{\alpha}^{G}$$

<u>for example :</u>



$$S_{k}^{T} = S_{G}^{T} * S_{k}^{G}$$

$$T.F = \frac{G}{1+GH}$$

① $$S_{G}^{T} = \frac{dT}{dG} * \frac{G}{T} = \frac{1+GH-GH}{(1+GH)^2} * \frac{G}{\frac{\cancel{G}}{\cancel{1+GH}}}$$

$$= \boxed{\frac{1}{1+GH}}$$

② $$S_{k}^{G} = \frac{dG}{dk} * \frac{k}{G}$$

$$G = \frac{k}{s+1}$$

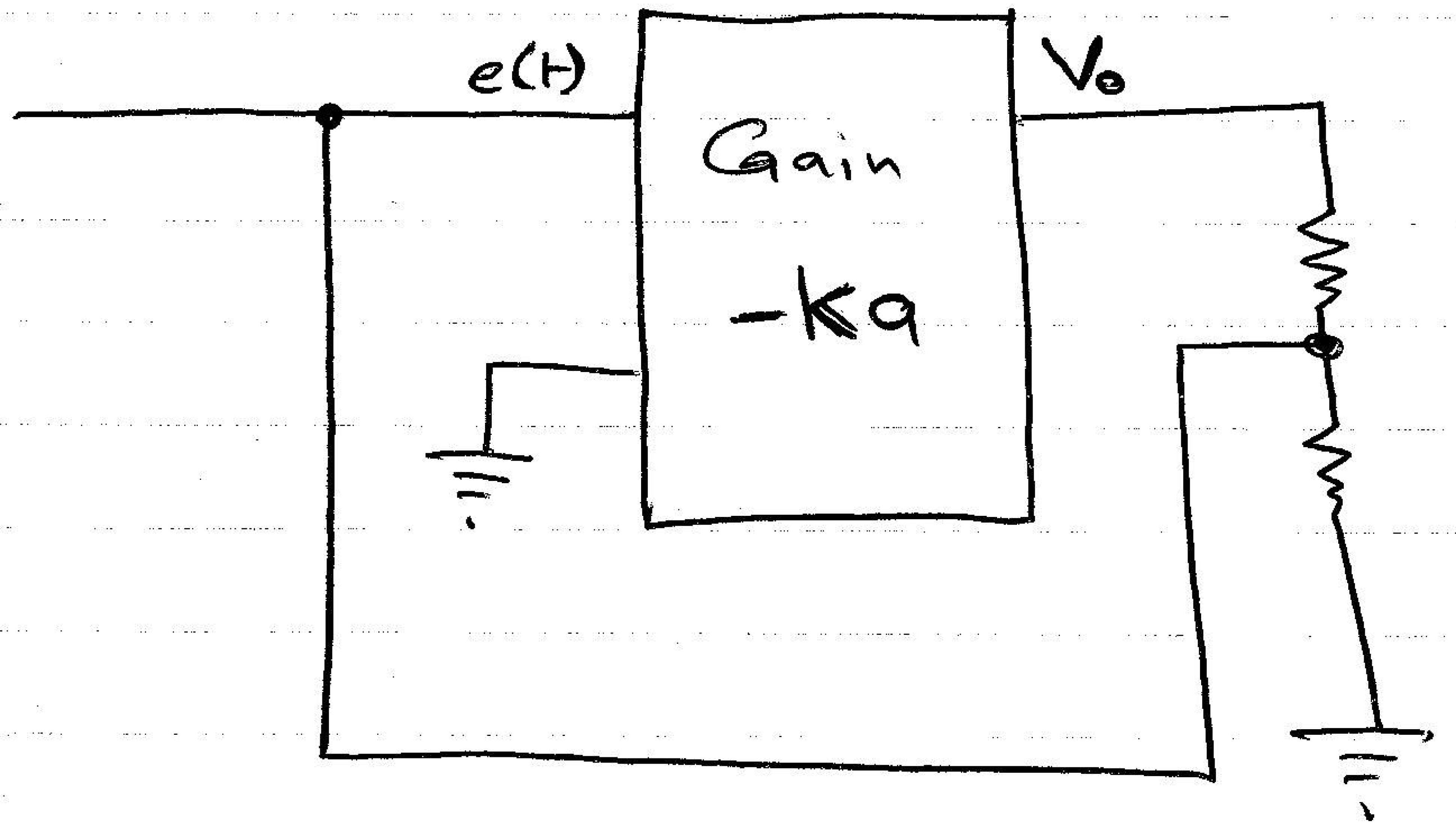
$$\therefore S_{k}^{G} = \frac{dG}{dk} * \frac{k}{G} = \frac{1}{\cancel{s+1}} * \frac{k}{\frac{\cancel{k}}{\cancel{s+1}}} = 1 * 1 = \boxed{1}$$

$$\Rightarrow \therefore S_{k}^{T} = S_{G}^{T} * S_{k}^{G} = \frac{1}{1+GH} * 1 = \boxed{\frac{1}{1+GH}}$$

Example :- Feed back Amplifier

المطلوب :-

Find the sensitivity of the system to $\boxed{ka}$ $\boxed{S^{T}_{ka}}$



$$\beta = \frac{R_2}{R_1}$$

$$e(t) = V_{in} + \frac{V_o * R_2}{R_1 + R_2} \quad \text{——— } ①$$

$$V_o = e(t) * -Ka \quad \text{——— } ②$$

$$e(t) = V_{in} + \frac{V_o * R_2}{R_1 + R_2} \quad ; \quad \boxed{\text{divide } \frac{V_o * R_2}{R_1 + R_2} \text{ on } R_1}$$

$$\therefore \quad e(t) = V_{in} + \frac{V_o \beta}{1 + \beta} \quad \text{——— } ① \; * *$$

$$V_o = e(t) * -ka \quad \text{——— } ②$$

from ① and ② :-

$$V_o = \left[V_{in} + \frac{\beta}{\beta + 1} V_o \right] * -Ka$$

$$\Rightarrow V_o = -Ka\, V_{in} - \frac{\beta}{\beta+1} Ka\, V_o$$

$$\Rightarrow V_o + \frac{\beta}{\beta+1} Ka\, V_o = -Ka\, V_{in}$$

$$\Rightarrow \boxed{\frac{V_o}{V_{in}} = \frac{-Ka}{1+\frac{\beta}{\beta+1} Ka}}$$

$$\boxed{\therefore\ T = \frac{-Ka}{1+\frac{\beta}{\beta+1} Ka}}$$

$$S_{Ka}^{T} = \frac{dT}{dKa} * \frac{Ka}{T}$$

$$\frac{dT}{dKa} = \frac{\left(1+\frac{\beta}{\beta+1} Ka\right)(-1) - (-Ka)\left(\frac{\beta}{\beta+1}\right)}{\left(1+\frac{\beta}{\beta+1} Ka\right)^2}$$

$$\therefore S_{Ka}^{T} = \frac{\left(1+\frac{\beta}{\beta+1} \cancel{Ka}\right)(-1) + \cancel{(Ka)\left(\frac{\beta}{\beta+1}\right)}}{\left(1+\frac{\beta}{\beta+1} Ka\right)^{\cancel{2}}} * \frac{\cancel{Ka}}{\frac{-\cancel{Ka}}{1+\frac{\beta}{\beta+1} Ka}}$$

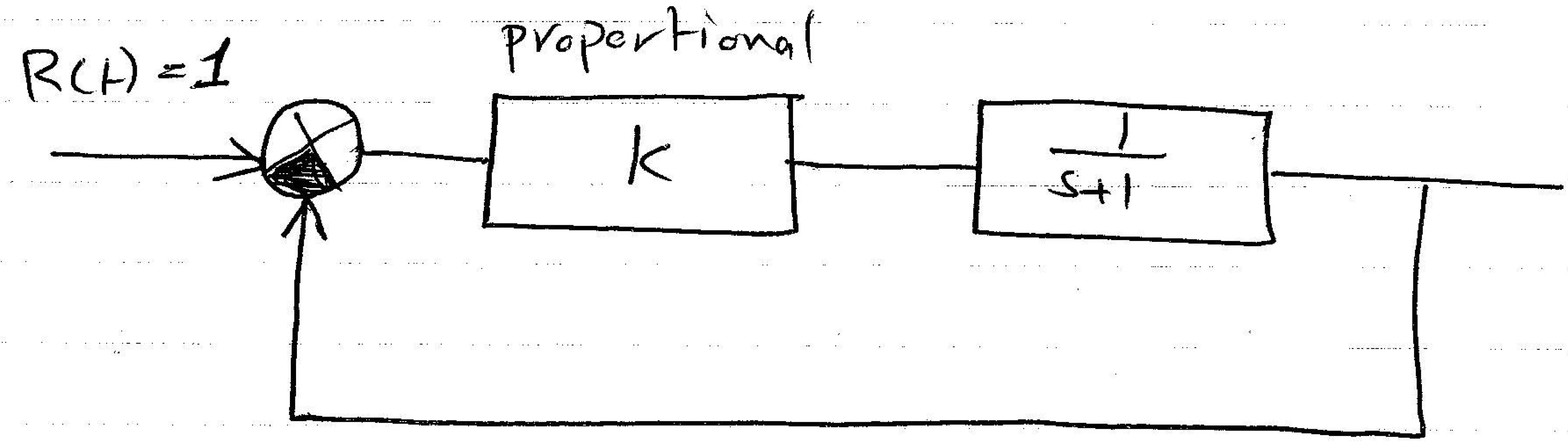
$$= \boxed{\frac{1}{1+\frac{\beta}{\beta+1} Ka}}$$

Ex :- for the following example , find the response of the system as k varies from 10 to 11

10.1

9.9

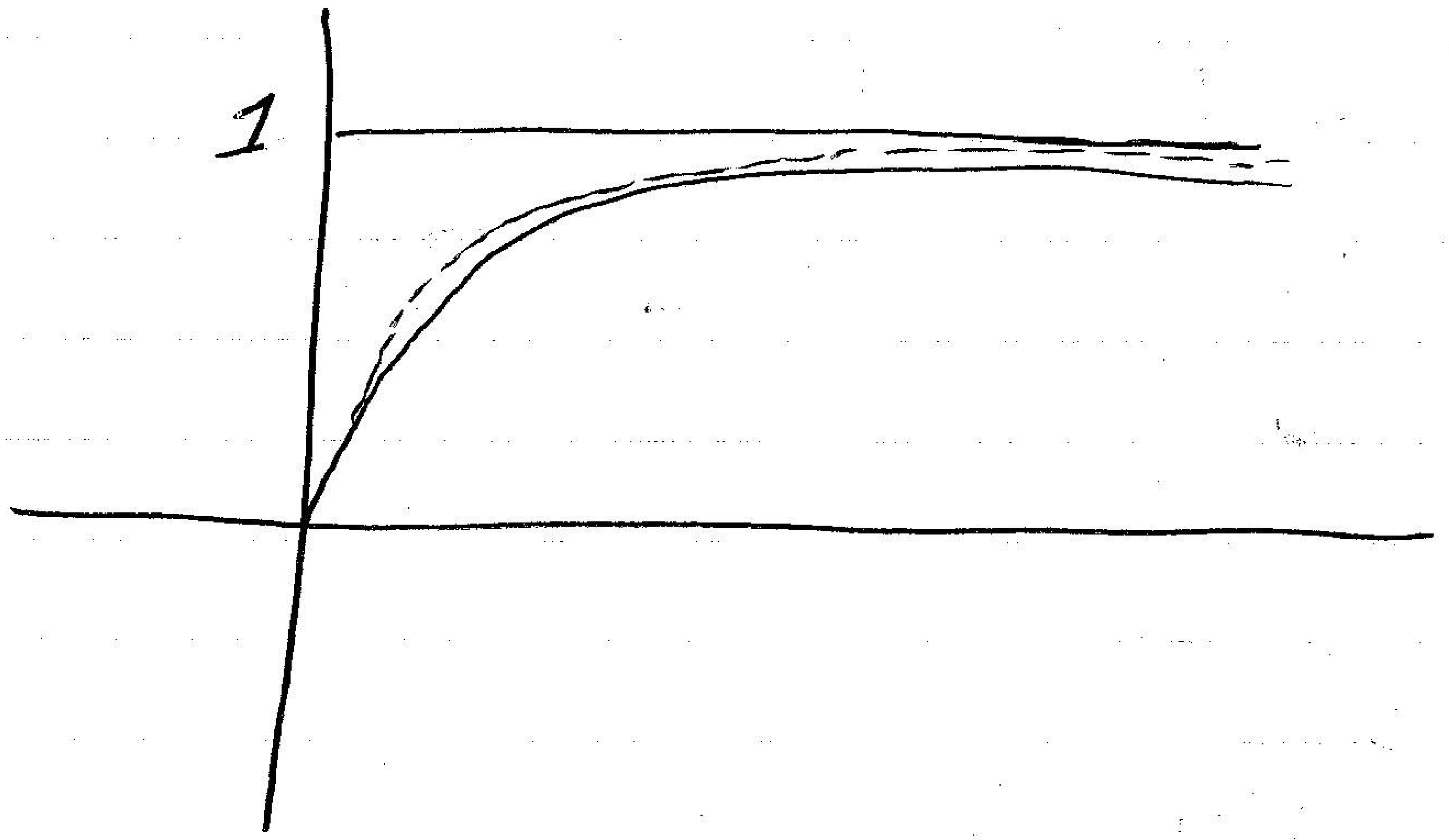
H.W



$$y_{ss} = \frac{k}{1+k} R$$

$$y_{ss}\Big|_{k=10} = \frac{10}{1+10} * 1 = \frac{10}{11} = \boxed{0.909}$$

$$y_{ss}\Big|_{k=11} = \frac{11}{1+11} * 1 = \frac{11}{12} = \boxed{0.916}$$



$$S_k^T = \frac{dT}{dk} * \frac{k}{T}$$

$$T(s) = \frac{kG}{1+kG}$$

$$\therefore S_k^T = \frac{dT}{dk} * \frac{k}{T}$$

$$= \frac{(1+kG)G - kG*G}{(1+kG)^2} * \frac{k}{\frac{kG}{1+kG}}$$

$$= \boxed{\frac{1}{1+kG}} = \frac{1}{1+\frac{k}{s+1}} = \boxed{\frac{s+1}{s+1+k}}$$

* at static response $\Rightarrow s = 0$

$$S_k^T = \frac{0+1}{0+1+k} = \boxed{\frac{1}{1+k}}$$

 (إكمال حل السؤال من المحاضرة الماضية)

$$S_G^T = \frac{\frac{\Delta T}{T}}{\frac{\Delta G}{G}} \Rightarrow \frac{\Delta T}{T} = S * \frac{\Delta G}{G}$$

$$y = T * X$$

$$y_{new} = (T + \Delta T)\, X$$

$$y_n = y + \Delta T\, X$$

$$= X\, T \left(1 + \frac{\Delta T}{T}\right)$$

$$= Y \left(1 + \frac{\Delta T}{T}\right)$$

Back To the previous Question (Page 88) :-

$$S_K^T = \frac{1}{1+K}$$

$$\frac{\Delta T}{T} = S_K^T * \frac{\Delta K}{K}$$

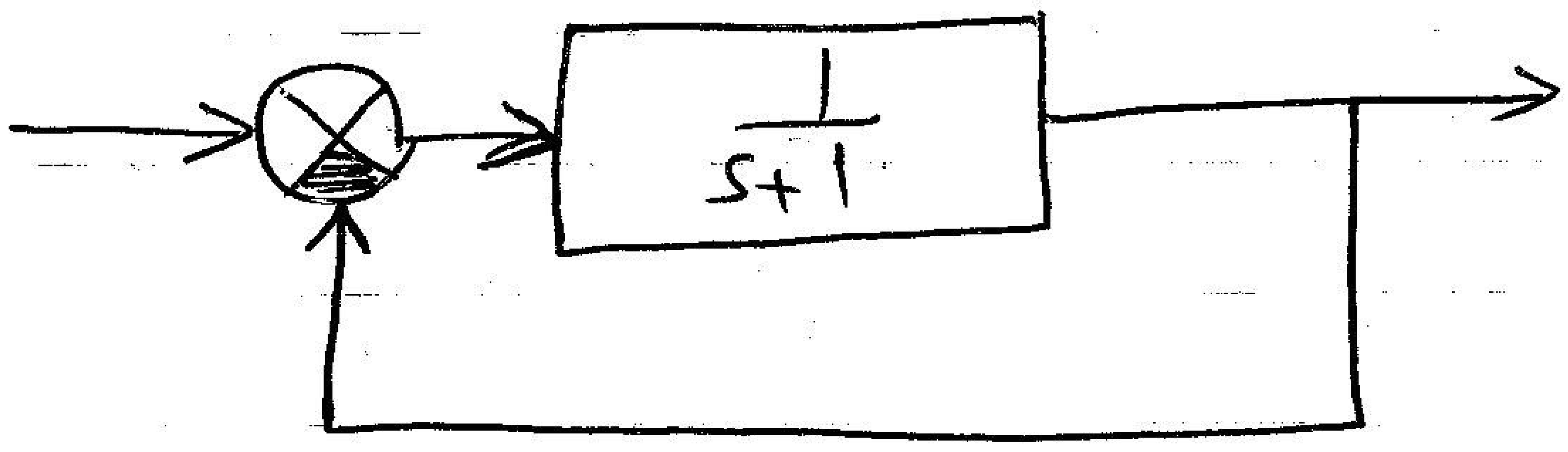
$$= \frac{1}{1+10} * \frac{1}{10} = \frac{1}{11} * \frac{1}{10} = \boxed{\frac{1}{110}}$$

$$y_n = y\left(1 + \frac{1}{110}\right)$$

$$= 0.909\left(1 + \frac{1}{110}\right) = \boxed{0.917}$$

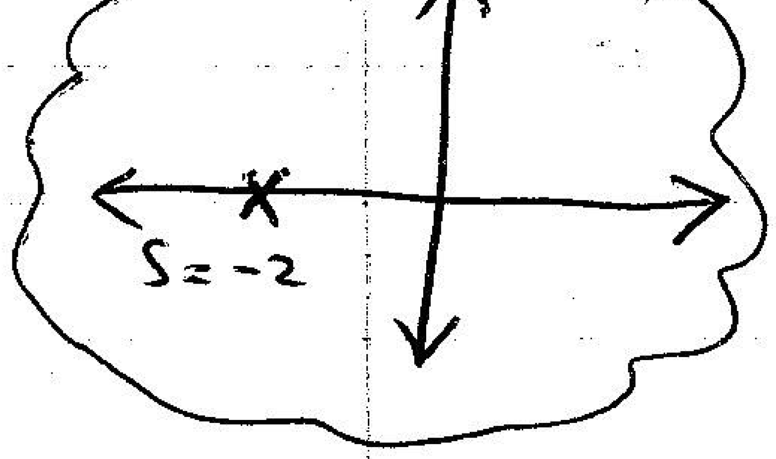
Root Loci

Ex

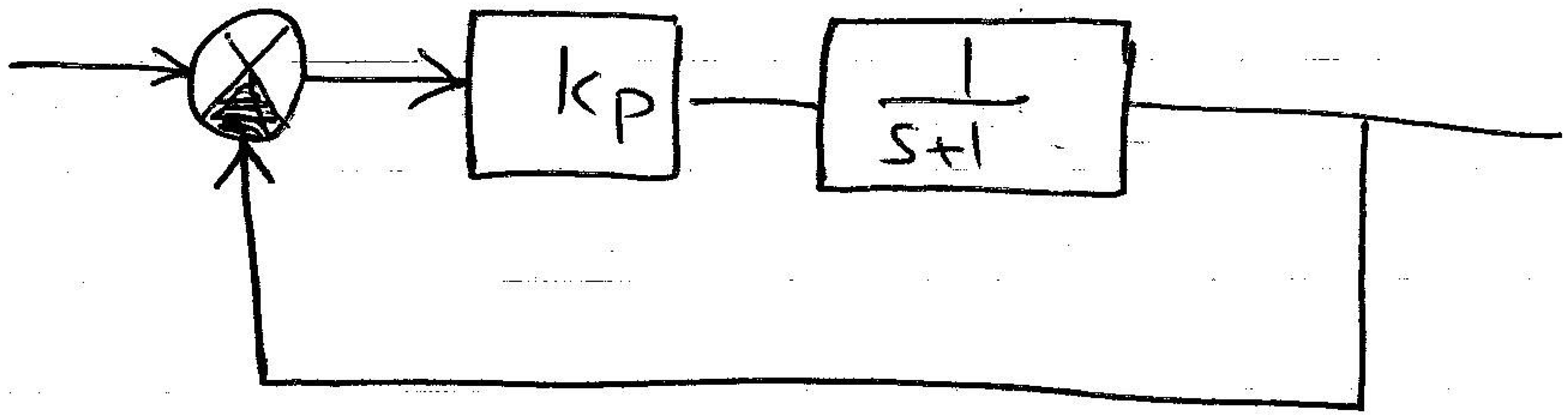


$$T.F. = \frac{1}{s+2}$$

$q(s) = s+2=0 \Rightarrow \boxed{S=-2}$ ∴ Stable



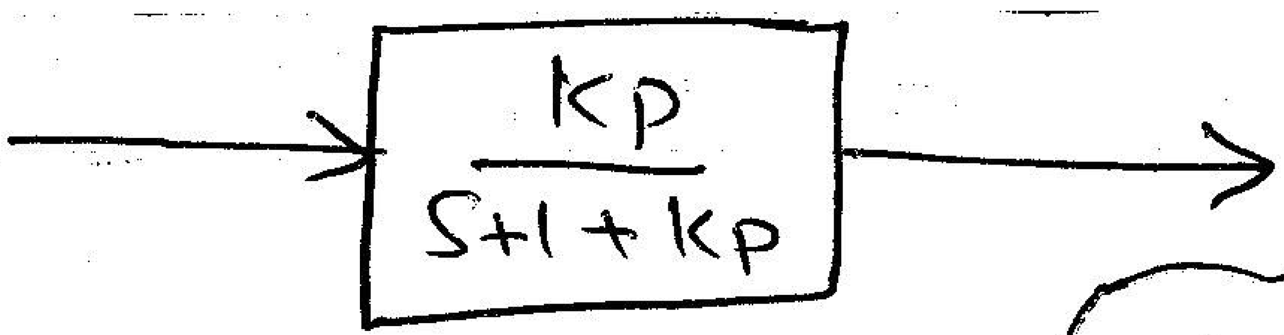
Ex



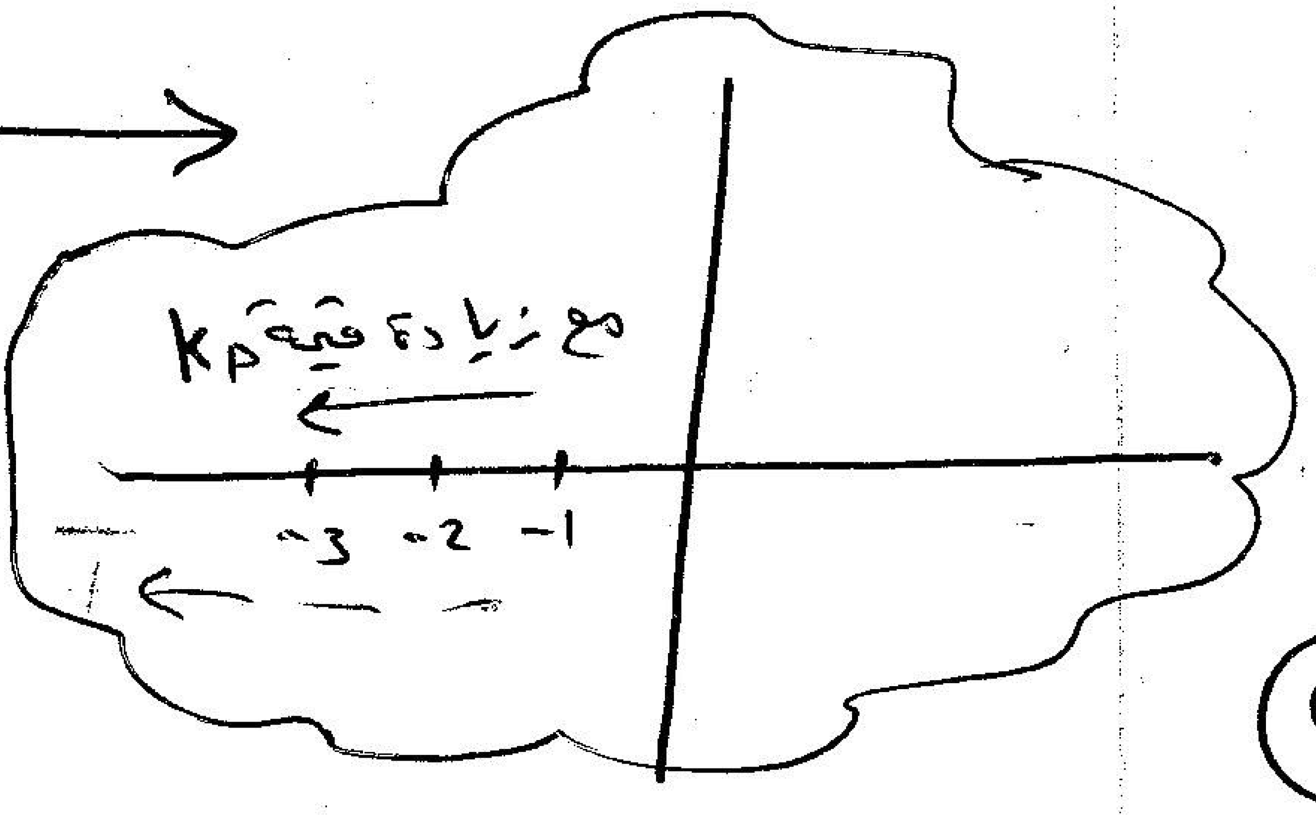
$\boxed{k_p = 0 \rightarrow \infty}$

$\Downarrow$

$\Downarrow$

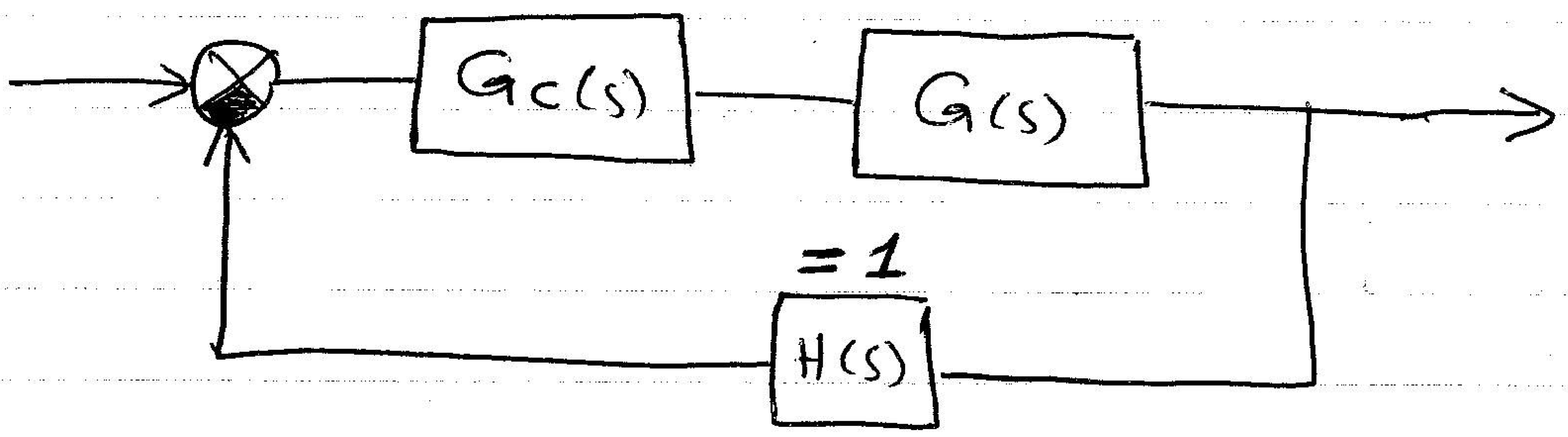


$\boxed{S = -1-k_p}$



Root Loci :- it determines how the roots of characteristic equation move around the S-plane as we change one parameter from $0 \rightarrow \infty$ by agraphical Method.

(*) This method was introduced by Evan in 1948



$$T(s) = \frac{G_c(s)\,G(s)}{1 + G_c(s)\,G(s)\,H(s)}$$

$$q(s) = 1 + G_c(s)\,G(s)\,H(s)$$

$$\boxed{q(s) = 1 + k\,G(s) = 0}$$

$$\Rightarrow k\,G(s) = -1$$

$$\Rightarrow k\,G(s) = -1 + j0 = |kG(s)| \angle kG(s) = -1 + j0$$

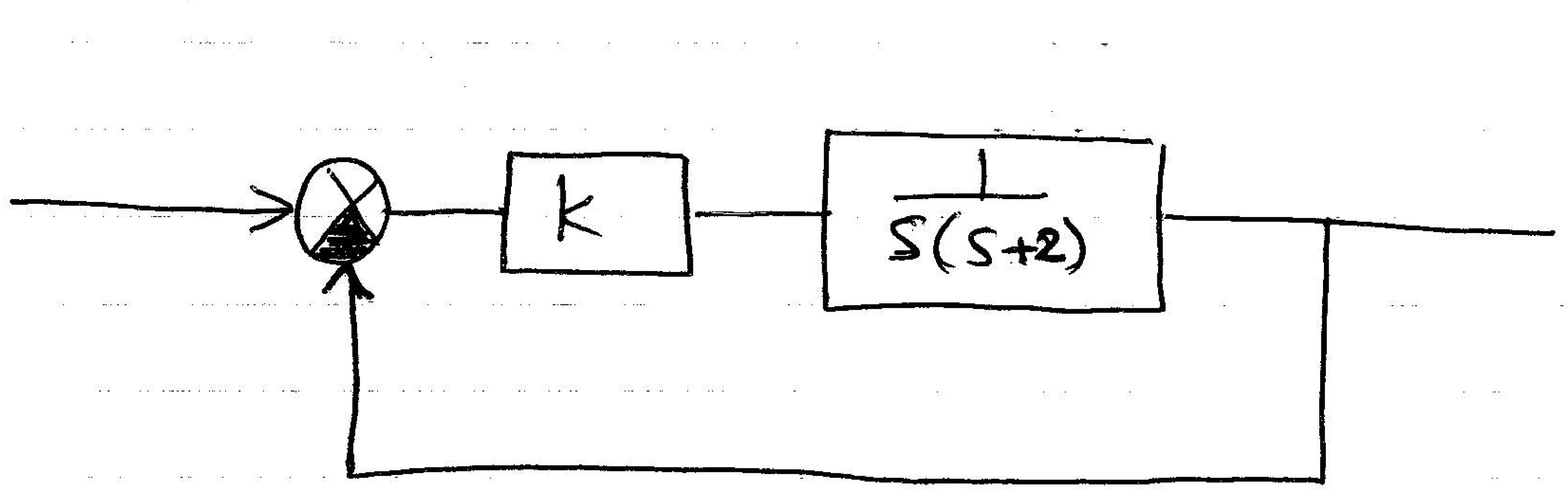
$$\boxed{|kG(s)| = 1}$$

$$\boxed{\angle kG(s) = 180 \mp k * 360} \quad \boxed{k = 1, 2, 3, 4 --}$$

$\Rightarrow$ We prefer to use angle -180°

$\circledast$ The root locus is the path of the roots of the ch. equ. traced out in S-plane as system parameter is changed from zero $\rightarrow$ infinity.

Ex Find the root locus of the following system.



$$T(S) = \frac{k}{S^2+2S+k}$$ as k varies from zero $\rightarrow \infty$

$$q(s) = S^2+2S+k = 0$$
$$= S^2 + 2\zeta W_n S + W_n^2 = 0$$

$$\boxed{S_{1,2} = -\zeta W_n \pm W_n \sqrt{\zeta^2 - 1}}$$

$\circledast$ We want to sketch the function [رسم تقريبي]

we will take some points :-

① $k=0 \Rightarrow q(S) = S^2+2S=0 \Rightarrow S(S+2) \Rightarrow S_{1,2} = 0, -2$

② $k = 0.5 \Rightarrow q(S) = S^2+2S+0.5=0$

$$\Rightarrow s_{1,2} = \frac{-2 \pm \sqrt{4-2}}{2} = -1 \pm \frac{1}{\sqrt{2}} = \boxed{-1 \pm 0.7}$$

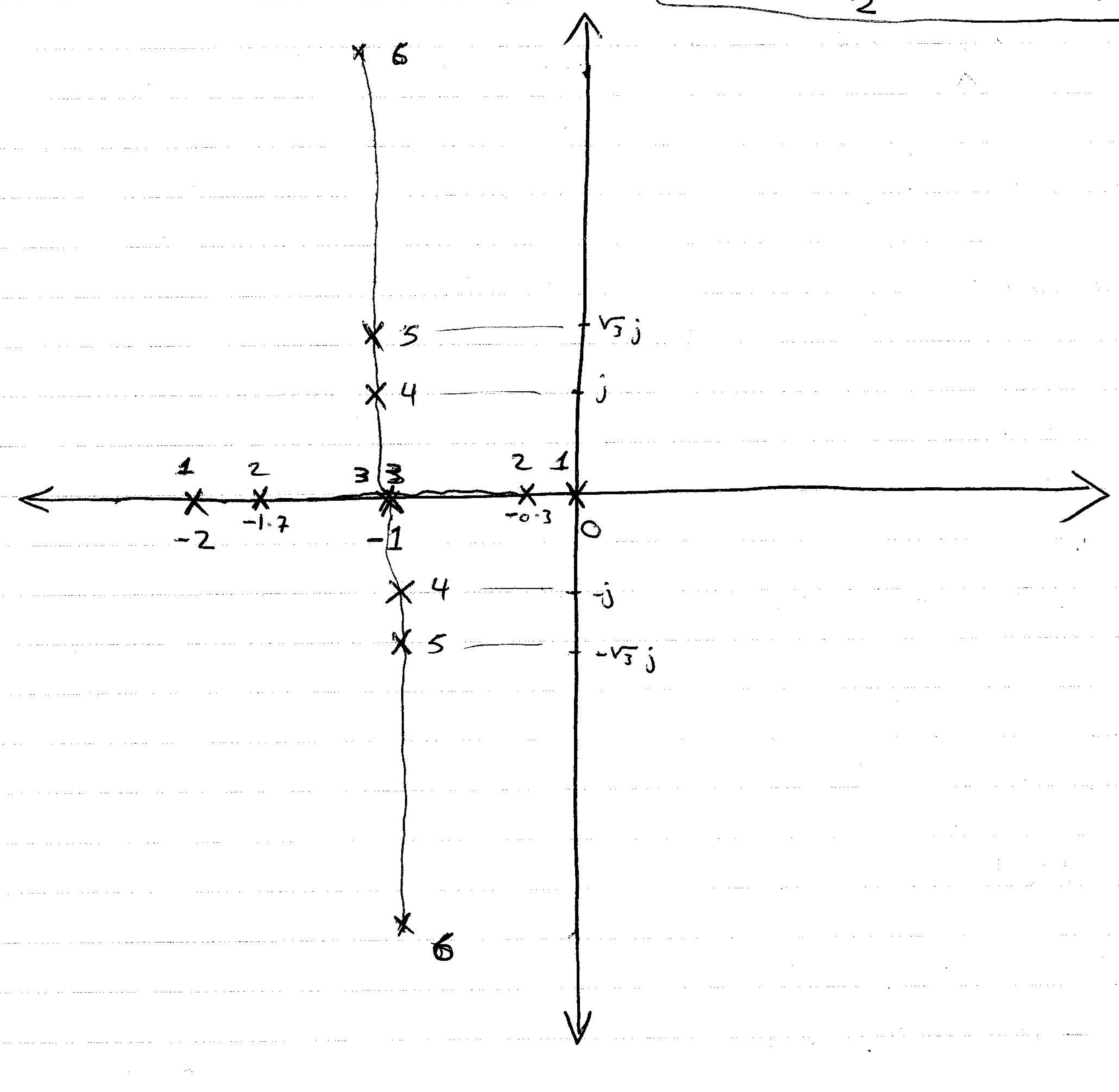
$$\Rightarrow -1 + 0.7 = \boxed{-0.3} \text{ and } -1 - 0.7 = \boxed{-1.7}$$

(3) $k = 1 \Rightarrow q(s) = s^2 + 2s + 1 = 0 \Rightarrow \boxed{s_{1,2} = \boxed{-1}}$

(4) $k = 2 \Rightarrow q(s) = s^2 + 2s + 2 = 0 \Rightarrow \boxed{s_{1,2} = -1 \pm j1}$

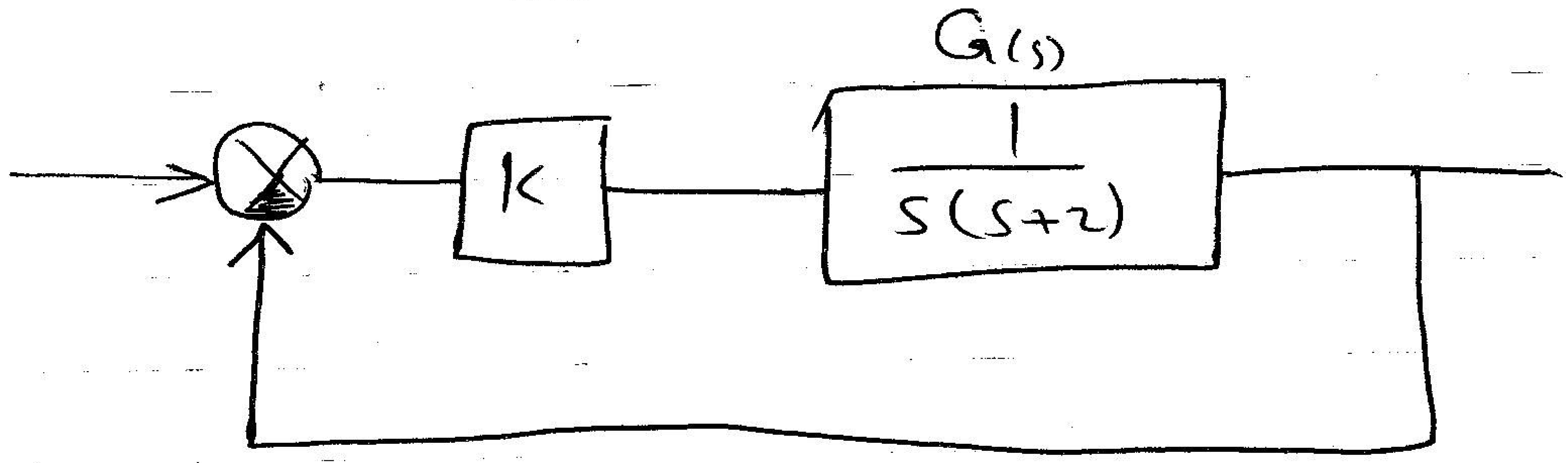
(5) $k = 4 \Rightarrow q(s) = s^2 + 2s + 4 = 0 \Rightarrow s_{1,2} = \boxed{-1 \pm j\sqrt{3}}$

(6) $k = \infty \Rightarrow q(s) = s^2 + 2s + \infty \Rightarrow \boxed{s_{1,2} = \frac{-2 \pm \sqrt{4-\infty}}{2} = -1 \pm j\infty}$



الاحد 12/22

Back to previous example :-



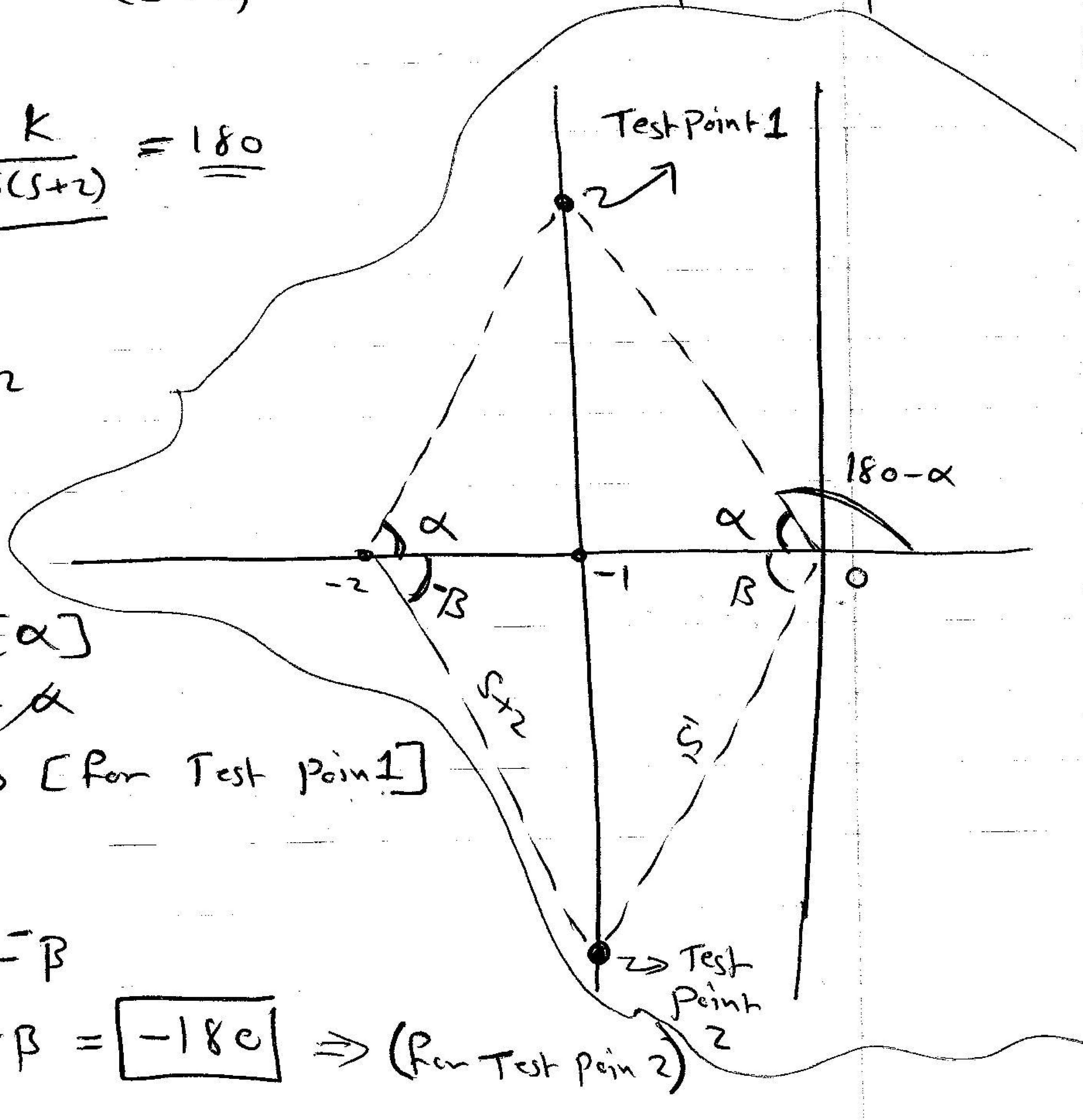
$$T(s) = \frac{\frac{K}{s(s+2)}}{\frac{K}{s(s+2)} + 1}$$

$$KG(s) + 1 = 0$$

$$\frac{K}{s(s+2)} + 1 = 0 \Rightarrow \frac{K}{s(s+2)} = -1$$

$$\Rightarrow \left|\frac{K}{s(s+2)}\right| = 1 \qquad \angle \frac{K}{s(s+2)} = 180$$

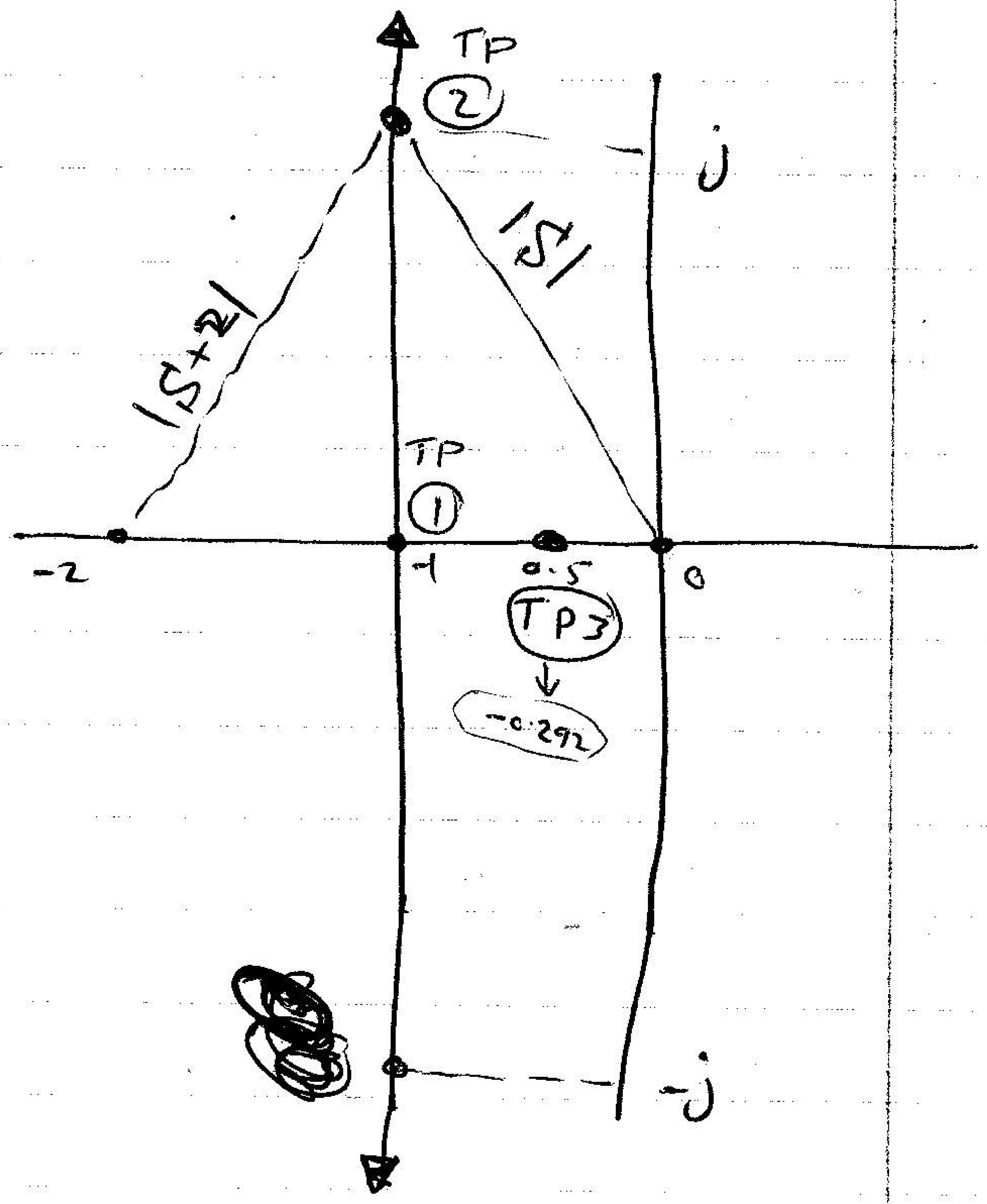
$$\boxed{180} = 0 - \angle s - \angle s+2$$



$$\text{angle} = 0 - [180-\alpha] - [\alpha]$$
$$= 0 - 180 + \cancel{\alpha} - \cancel{\alpha}$$
$$= \boxed{-180} \Rightarrow \text{[for Test point 1]}$$

$$\text{angle} = 0 - [180+\beta] - \beta$$
$$= -180 - \beta + \beta = \boxed{-180} \Rightarrow \text{(for Test point 2)}$$

$$\left|\frac{k}{s(s+2)}\right| = 1 \Rightarrow \frac{|k|}{|s|\,|s+2|} = 1 \Rightarrow \frac{k}{|s|\,|s+2|} = 1$$



① $\boxed{\text{at } k=1 \quad (\text{Test point } 1)(TP1)}$

$$\Rightarrow \frac{1}{|1|\,|1|} = \boxed{1}$$

② $\boxed{\text{at } \boxed{TP2} \quad \boxed{k=2}}$

$$\frac{k}{|s|\,|s+2|} = 1$$

$$|s| = \sqrt{1^2+1^2} = \sqrt{2}$$

$$|s+2| = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\Rightarrow \frac{k}{\sqrt{2} * \sqrt{2}} = \frac{2}{\sqrt{2}\sqrt{2}} = \frac{2}{2} = \boxed{1}$$

③ $\boxed{\text{at } \boxed{TP3} \quad \text{at } \boxed{k=0.5}}$

$$|s| = \boxed{0.292}$$

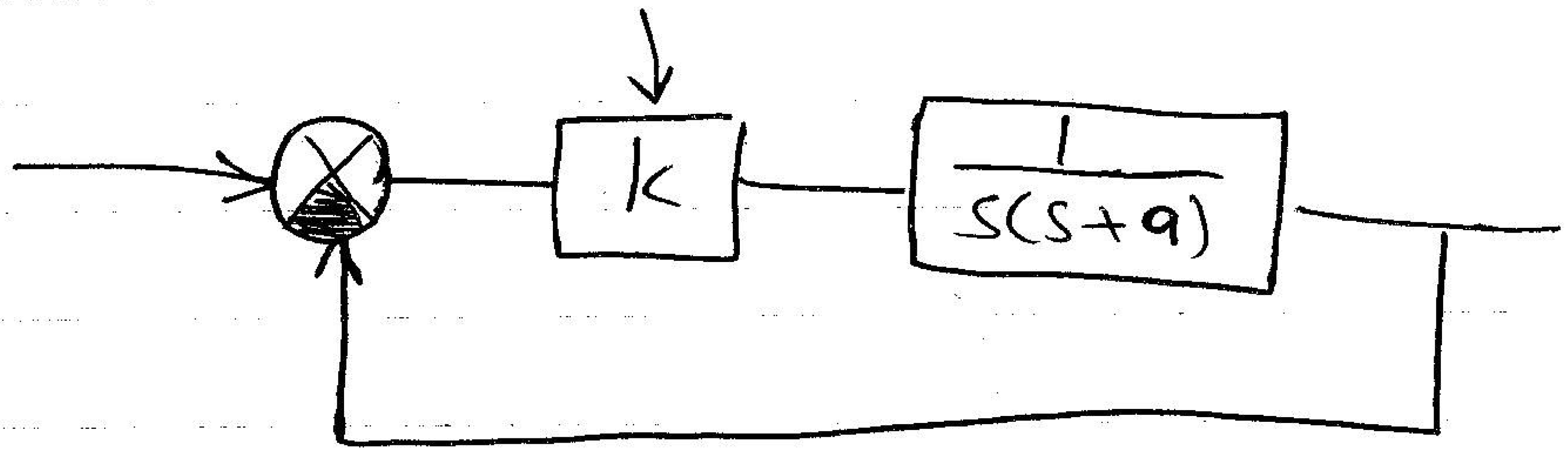
$$|s+2| = \boxed{1.708}$$

$$\Rightarrow \frac{k}{|s|\,|s+2|} = \frac{0.5}{|0.292|\,|1.708|} = 1.0025 \approx \boxed{1}$$

k is constant
a is Variable

EX

Constant = 1



$KG(s) = -1$

$|KG(s)| = 1$

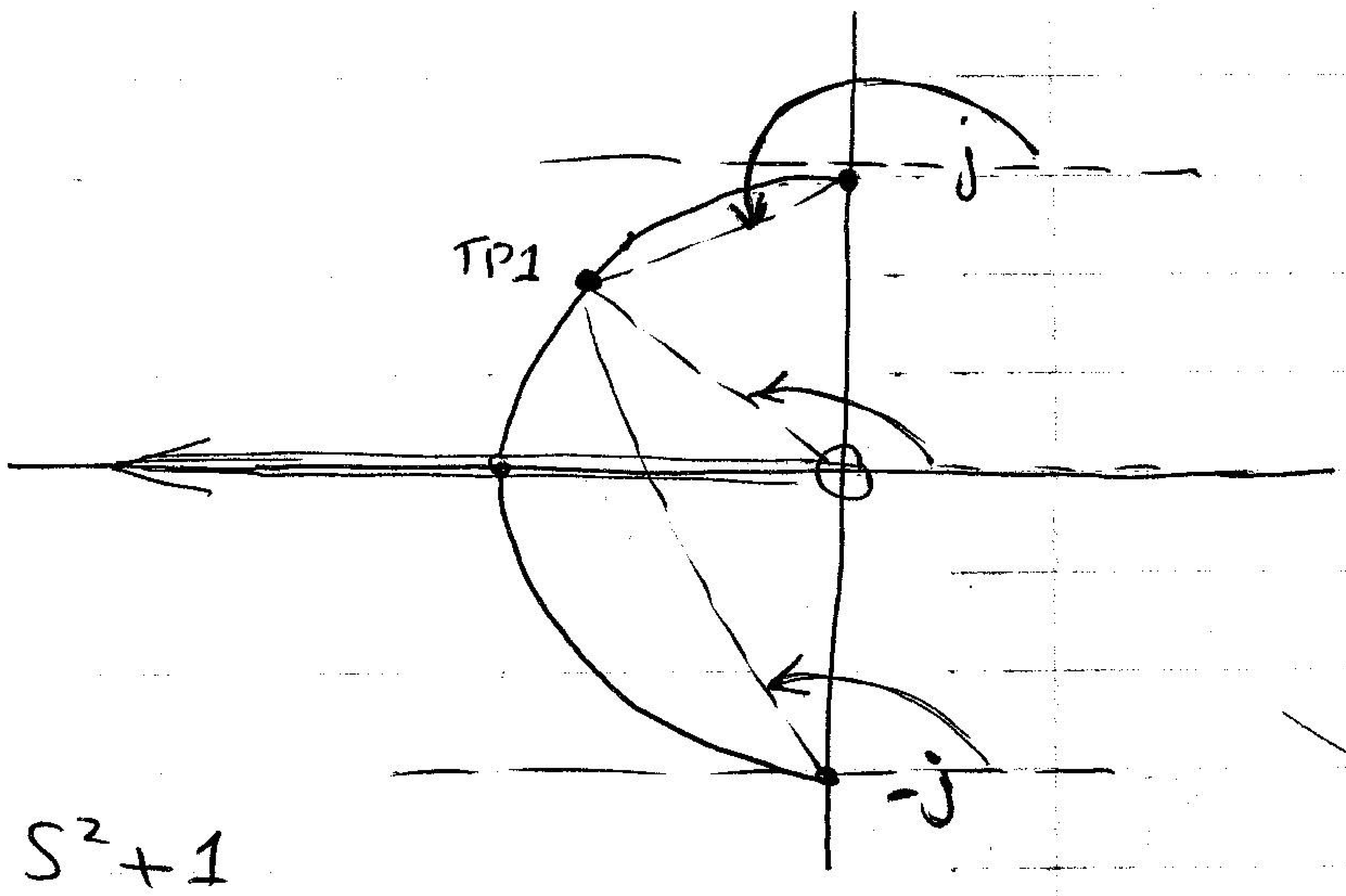
$\angle KG(s) = -180^\circ$

but a is the Variable

$aG(s) = -1$

$\angle aG(s) = -180^\circ$

$$\Rightarrow \frac{\frac{1}{s(s+a)}}{1+\frac{1}{s(s+a)}}$$



$$\Rightarrow \frac{1}{s(s+a)} + 1 = 0$$

$$1 + s^2 + sa = 0$$

$$as + s^2 + 1 = 0 \qquad \div\ s^2+1$$

$$a\frac{s}{s^2+1} + 1 = 0$$

$$\Rightarrow a\frac{s}{s^2+1} + 1 = 0$$

$$\left|\frac{as}{s^2+1}\right| = 1 \Rightarrow \frac{a\,|s|}{|s+j|\,|s-j|}$$

$$\angle\frac{as}{s^2+1} = -180^\circ \Rightarrow -180^\circ = |s| - |s+j| - |s-j|$$

"خطوات حل مسائل الـ Root Locus"

Root Locus Procedures

STEP 1 Write the characteristic equation as

$$1+F(s)=0$$

Then arrange the equation so that the parameter of interest appears as the multiplying factor $\boxed{1+Kp(s)=0}$

note : in most cases , $\boxed{F(s) \neq Kp(s)}$

$$1+\frac{s}{s+a}=0 \quad \rightarrow F(s)$$

$$s+a+s=0$$

$$2s+a=0 \quad /2s$$

$$1+\frac{a}{2s}=0$$

$$1+a\,p(s)$$

STEP 2 Write the polynomial in the form of Poles & Zeros

$$\boxed{1+K\frac{\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)}}$$

STEP 3 Locate the poles & Zeros on the S-plane with selected symbols as K varies from $0 \rightarrow \infty$

pole	X
zero	O

إثبات النتيجة

Ex Rewrite the equation in the following form :-

$$\prod_{j=1}^{n} (S + P_i) + K \prod_{i=1}^{m} (S + Z_i)$$

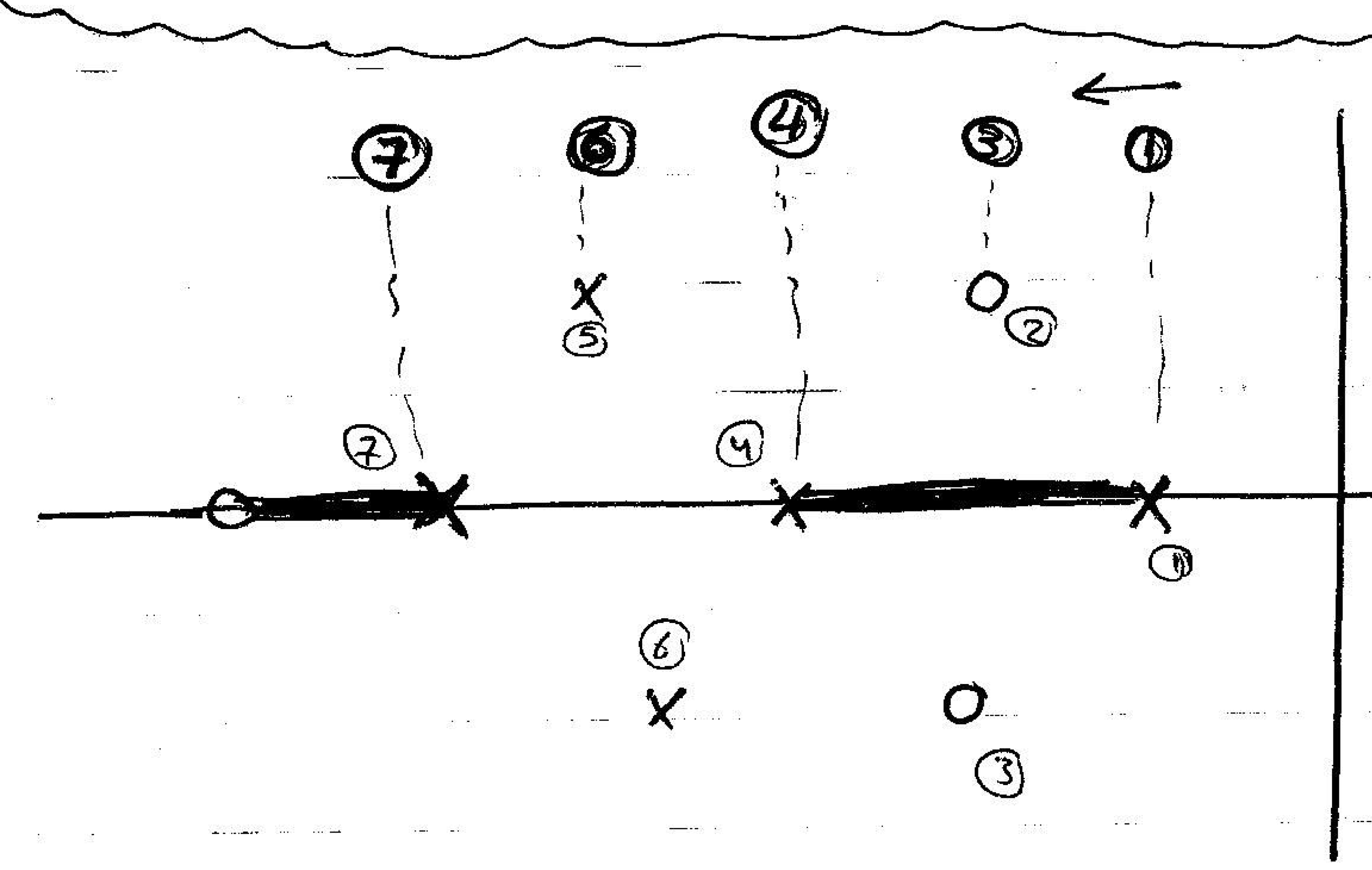
$\boxed{\text{when } K = 0}$ ⟹ The roots of ch. equ. are simply the poles of $P(s)$.

$\boxed{\text{when } K = \infty}$ ⟹ The roots of ch. equ. are simply the zeros of $P(s)$

⊛ There is $n - m$ branches of the root Locus approaches ∞

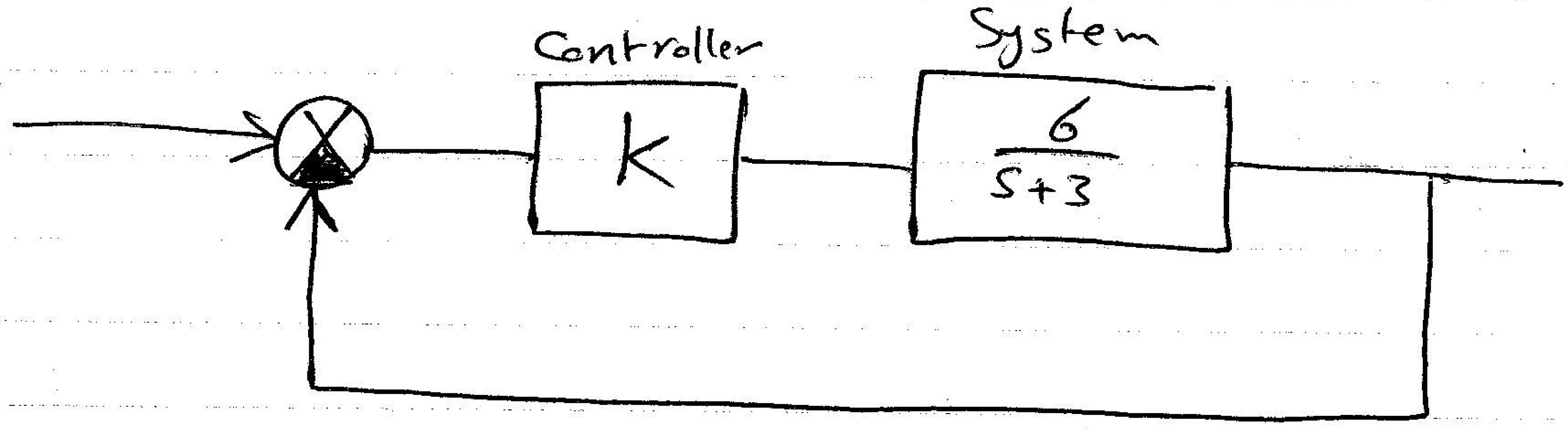
$\boxed{\text{STEP 4}}$ Locate the segment of the real-axis that are root Loci

" The root Locus on the real-axis always Lies in a section of the real-axis to the Left of an odd number of poles & zeros "



نبدأ من اليسار (يجب أن نجمع الـ Poles والـ Zeros) اذا وجدنا العدد فردي، يكون هناك رسم Root locus بعده، نضل مستمرين في السير من اليمين الى اليسار، اذا اصبح مجموع الاقطاب والاصفار زوجي، لا يكون هناك رسم بعده، وهكذا.

Example :- plot the root Locus of the following system as k varies from zero to ∞.



$$T(s) = \frac{\frac{6k}{s+3}}{1 + \frac{6k}{s+3}}$$

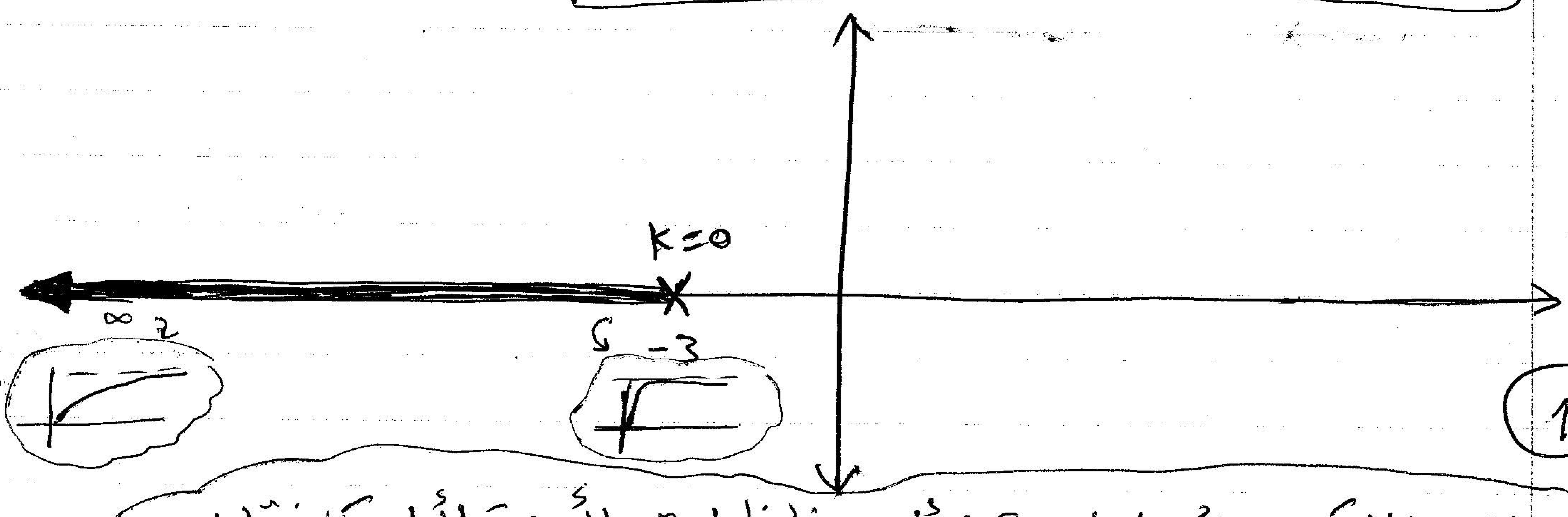
STEP 1 $\quad \boxed{1 + \frac{6k}{s+3} = 0} \equiv \boxed{1 + F(s) = 0}$

STEP 2 $\quad \boxed{1 + K\frac{6}{s+3} = 0}$ ← جاهزة هنا صدفة زبطت

STEP 3 $\quad \boxed{1 + K\frac{6}{(s+3)}}$

STEP 4 $\quad$ Poles ⇒ $\boxed{s = -3}$

Zeros ⇒ Not available, but there is one goes to ∞

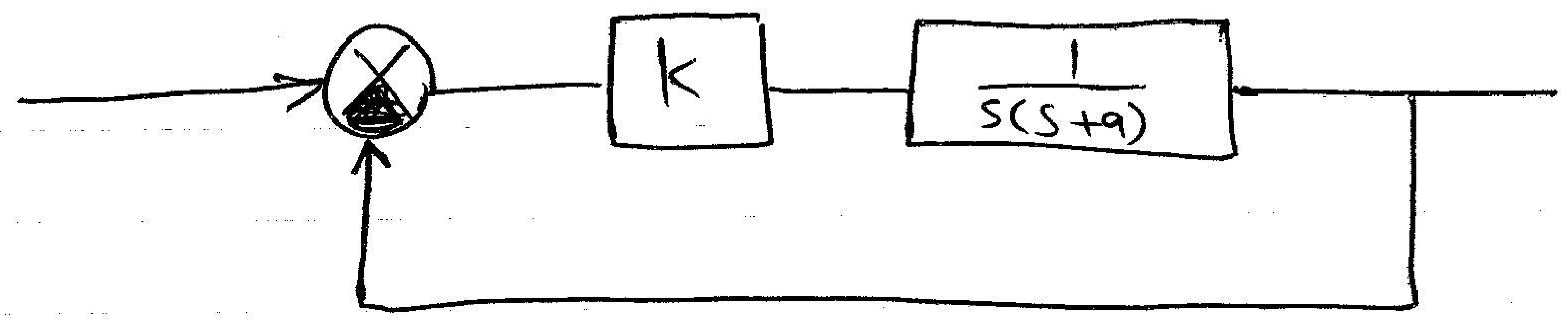


ملاحظة :- اذا كان نظام من الدرجة الأولى ، فالخطوات الأربعة الأولى كافية لرسم Root Locus.

الخميس 12/26

Ex :-

as $a = 0 \rightarrow \infty$



$$T(s) = \frac{\frac{k}{s(s+a)}}{1 + \frac{k}{s(s+a)}}$$

$\boxed{\text{STEP 1}}$: $1 + F(s) = 0$

$$1 + \frac{k}{s(s+a)} = 0$$

$\boxed{\text{STEP 2}}$: $1 + \frac{k}{s(s+a)} = 0$ $\quad$ $1 + a\,p(s) = 0$

$$s(s+a) + k = 0$$

$$s^2 + as + k = 0$$

$$s^2 + k + as = 0 \qquad \div (s^2 + k)$$

$$\Rightarrow \boxed{1 + \frac{as}{s^2 + k}} \rightarrow 1 + a\,p(s) = 0$$

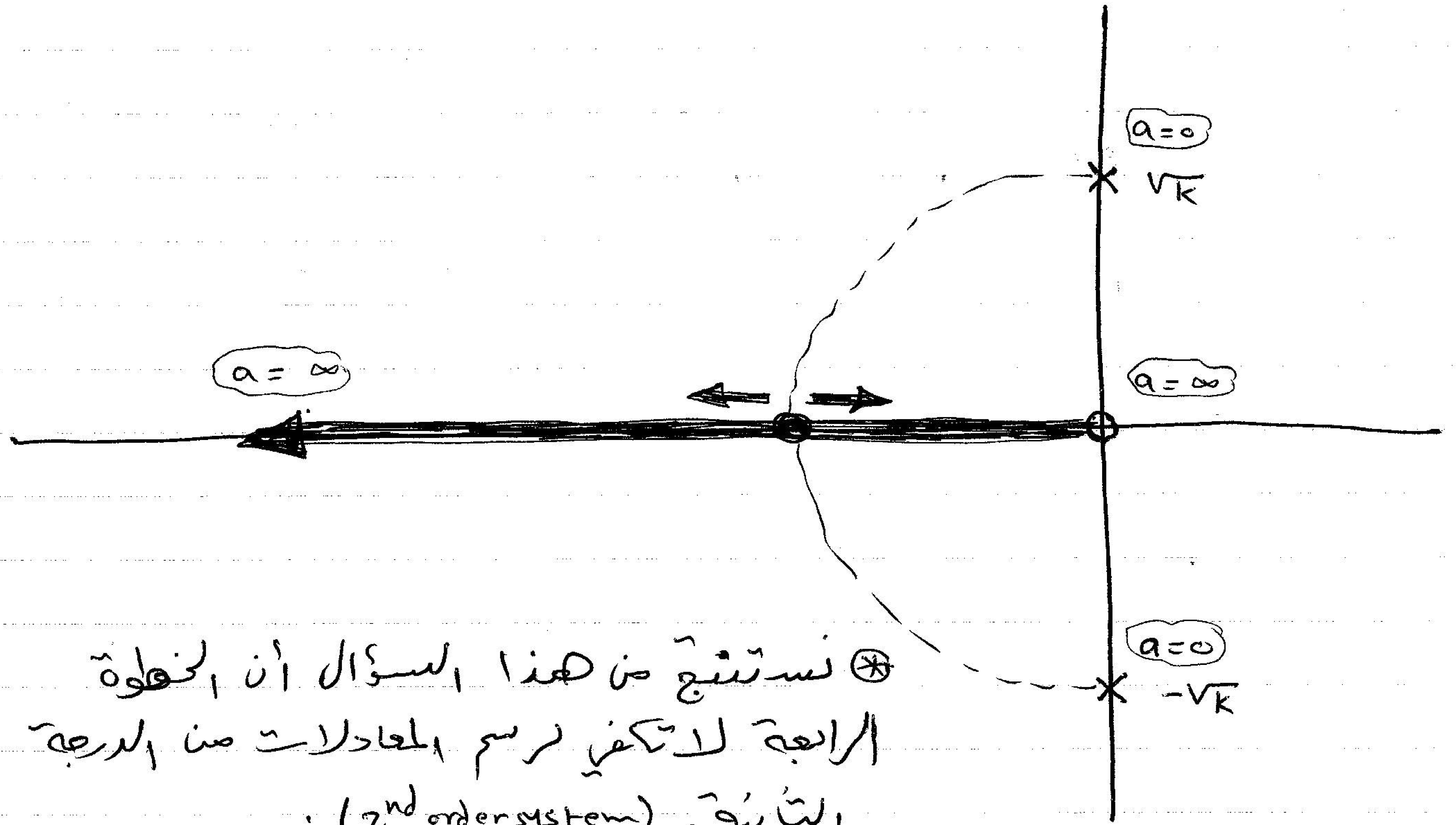
أصبح على الصورة القياسية

$\boxed{\text{STEP 3}}$: $1 + a\frac{s}{s^2 + k} = 0 \Rightarrow 1 + a\frac{s}{(s + j\sqrt{k})(s - j\sqrt{k})} = 0$

$\boxed{\text{STEP 4}}$:- $\boxed{\text{Poles}}$ $s = \pm j\sqrt{k}$

$\boxed{\text{Zeros}}$ $s = 0$

Pole conjugate always symmetrical



* نستنتج من هذا السؤال أن الخطوة الرابعة لا تكفي لرسم المعادلات من الدرجة الثانية. (2^{nd} order system)

Back To General STEPS :- [عودة للخطوات الرئيسية]

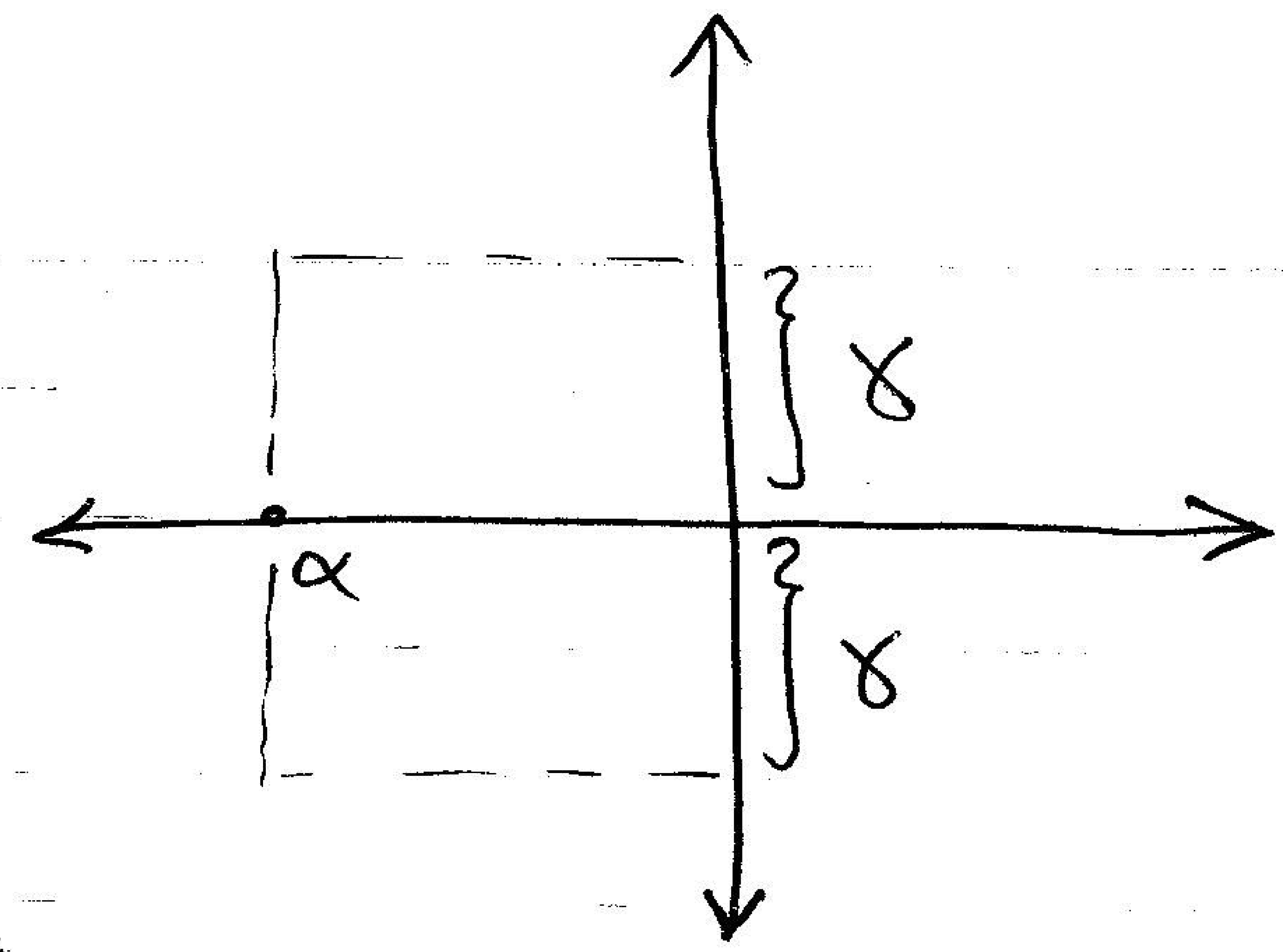
STEP 5 Determine the number of separate loci

$\boxed{N_{\underline{o}} \text{ of Separate Loci} = N_{\underline{o}} \text{ of Poles}}$

STEP 6 The root Loci must be symmetrical with respect to the horizontal real-axis, because the complex roots must appear as pole conjugate.

$$\boxed{S_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b}{2a} \pm \frac{j\,q}{2a} = \alpha \pm j\gamma}$$

$\boxed{\alpha + j\gamma}$ →



STEP 7 The Loci proceds to the zero at ∞ along asymptote centered at σ_A, and with angle ϕ_A, when the No of ~~poles of P(s)~~ finite zeros of P(s) is less than the No of Poles of P(s)

$\boxed{N_z < N_p}$ Then we have No of zeros at ∞

$$\boxed{N = N_p - N_z}$$

These Linear asymptotes are centered at a point on the real axis & given by :-

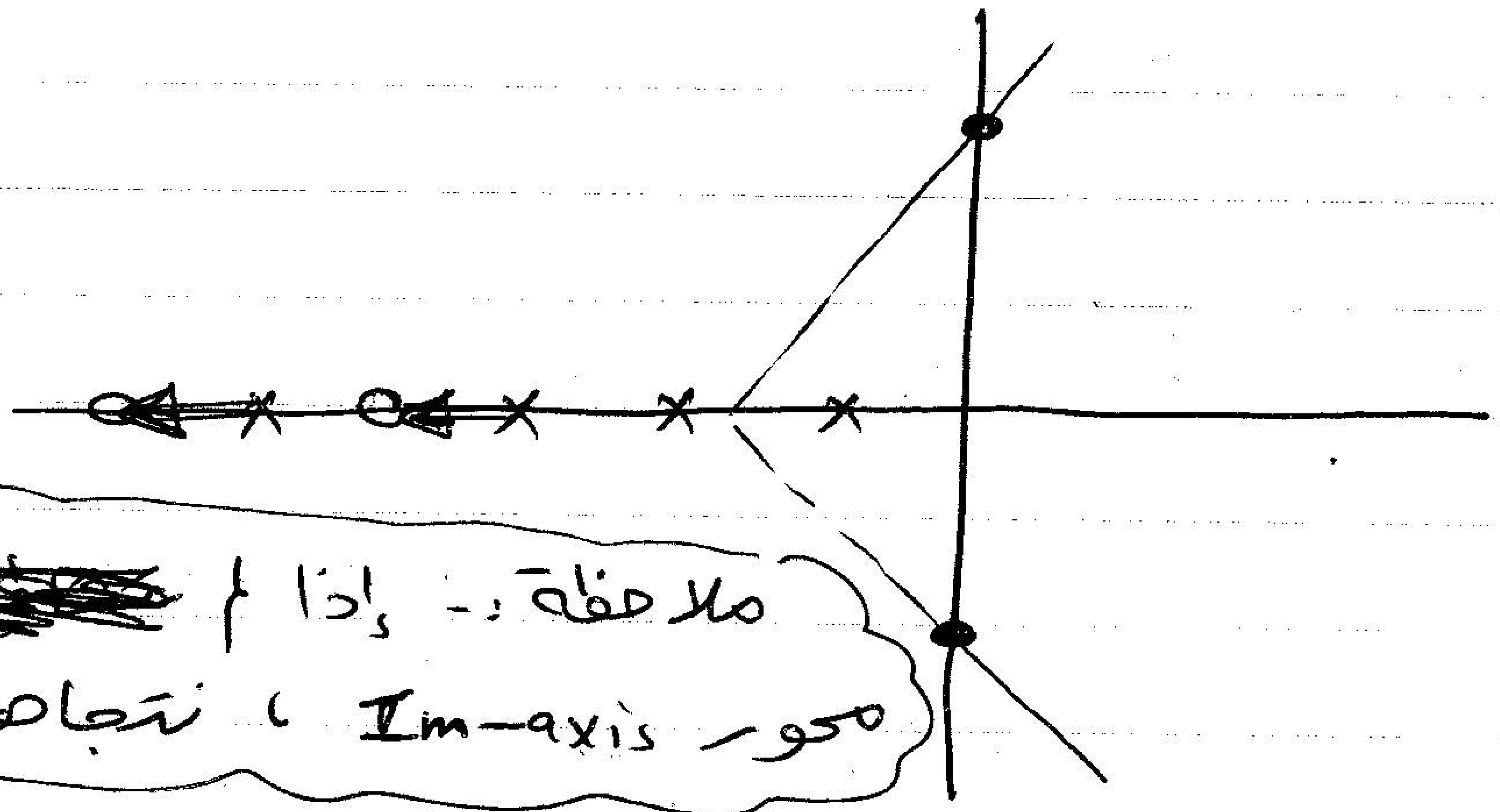
$$\boxed{\sigma_A = \frac{\sum \text{Poles of } P(s) - \sum \text{Zeros of } P(s)}{N_p - N_z}}$$

* The angle of asymptotes with respect to real axis is given by :

$$\boxed{\phi_A = \frac{2q + 1}{N_p - N_z} * 180} \quad q = (0, 1, 2, 3 \text{ — } N_p - N_z - 1)$$

STEP 8

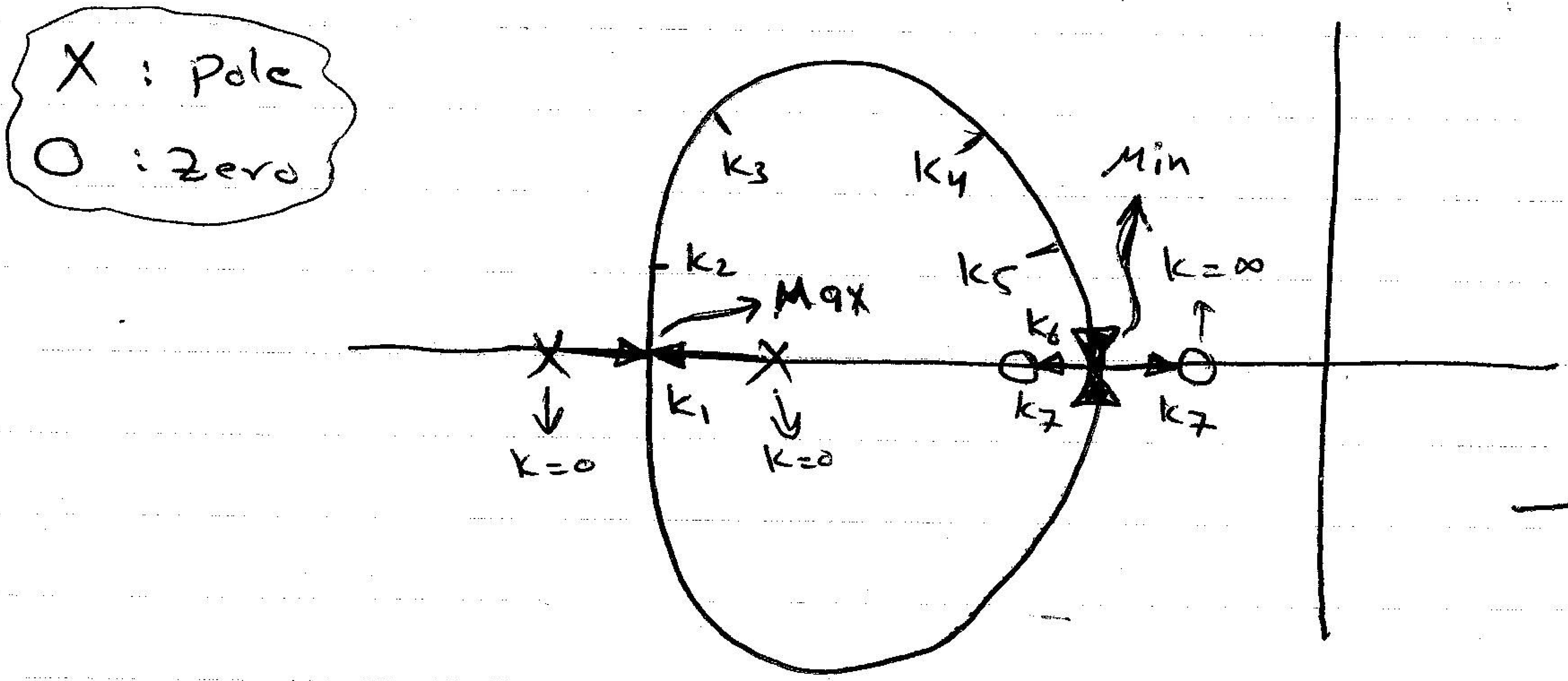
Determine the point of which the root Locus crosses the Imaginary-axis (use Routh Hurwitz Criteria)

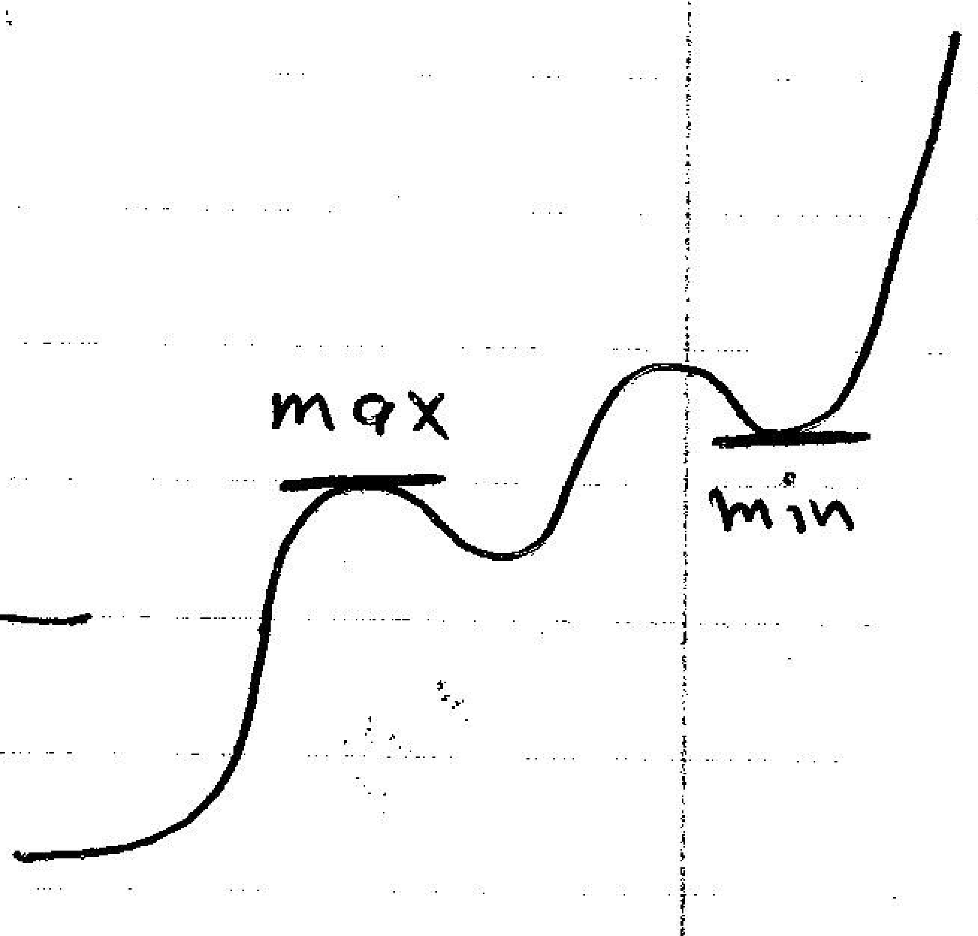


ملاحظة :- إذا لم يكن هناك تقاطع مع محور Im-axis ، نتجاهل الخطوة الحالية.

STEP 9

Determine the Break-away & Break-in points.





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① Put or find the equation of k

$$\boxed{1 + kP(s) = 0 \Rightarrow k = \frac{-1}{P(s)}}$$

② Find $\frac{dk}{ds} = 0$

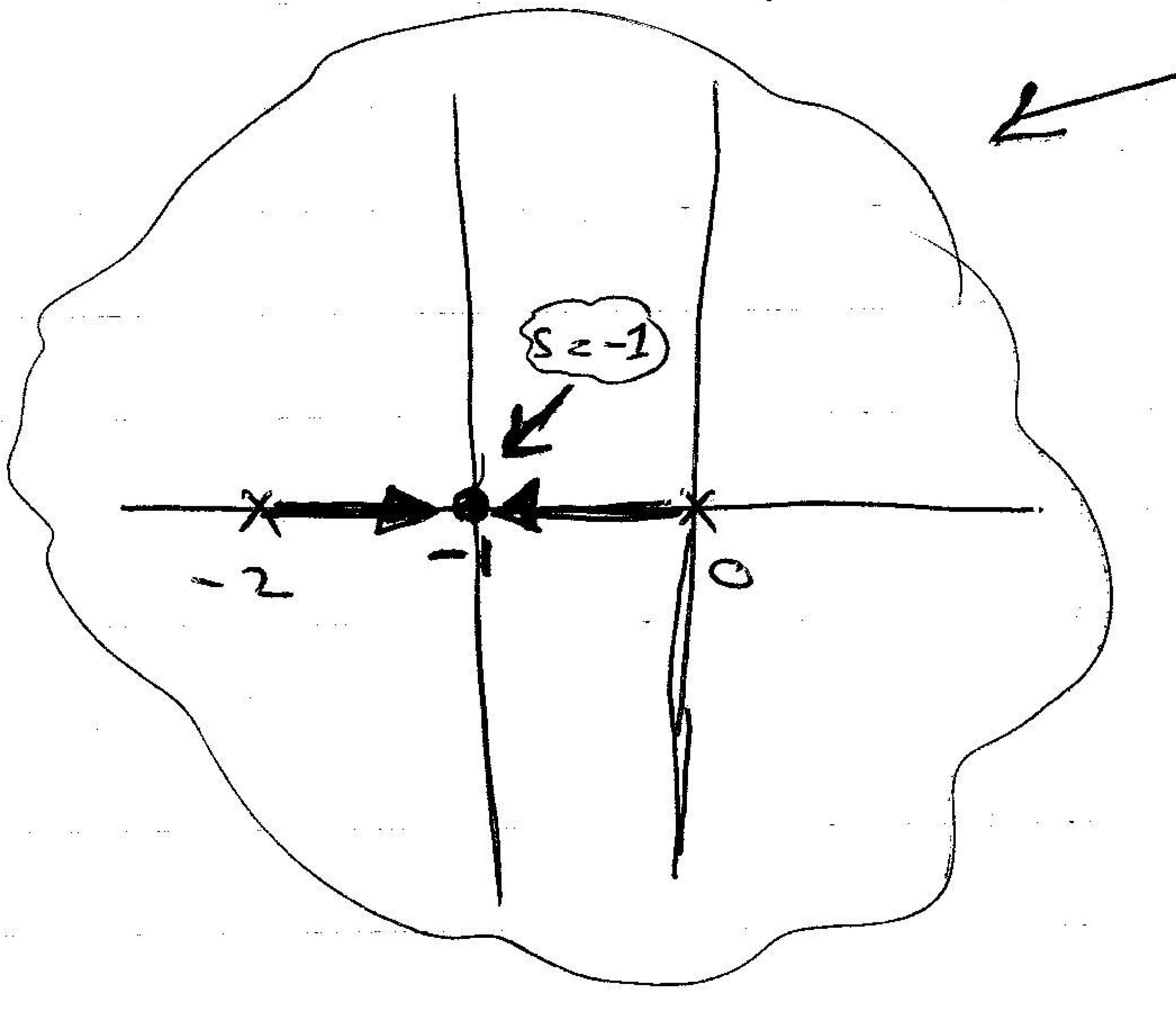
Ex

$$1+\frac{k}{s(s+2)}=0$$

$$k=-s(s+2)$$

$$k=-s^2-2s$$

$$\frac{dk}{ds}=-2s-2=0 \Rightarrow -2s=2 \Rightarrow \boxed{s=-1}$$

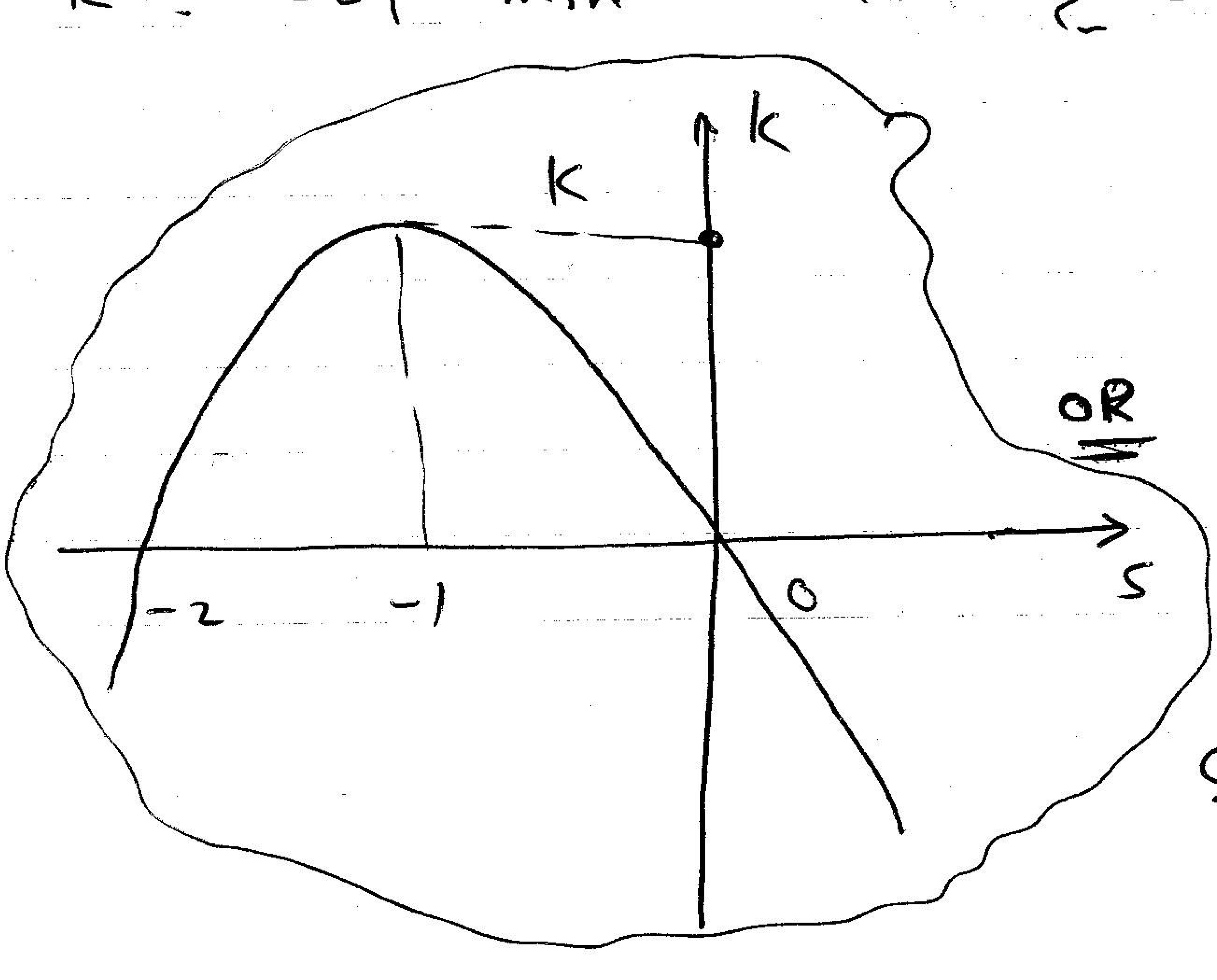


طرق حل الخطوة التاسعة

① ايجاد المشتقة $\frac{dk}{ds}=0$

② إعطاء Table لإيجاد قيم max ، min الخاصة بـ k

③ إعطاء منحنى لإيجاد قيم max ، min الخاصة بـ k



OR

s	0	-1	-2	-3
k	0	1	0.2	0.1

OR $\frac{dk}{ds}=0$

Ex find the root Locus of the following system

$$1 + \frac{k}{(S+2)(S+4)} = 0$$

- $\boxed{\text{STEP 1}}$ already done.

- $\boxed{\text{STEP 2}}$ already done.

$\boxed{\text{STEP 3}}$ Poles : $\boxed{S=-2}$ $\boxed{S=-4}$

$\boxed{\text{STEP 4}}$ determine the Loci Segment.

$\boxed{\text{STEP 5}}$ No of Loci $= \underline{\underline{2}}$.

$\boxed{\text{STEP 6}}$ will be taken in consideration.

$\boxed{\text{STEP 7}}$ $\sigma A = \frac{\sum \text{Poles} - \sum \text{Zeros}}{NP - NZ} = \frac{-2-4-(0)}{2-0}$

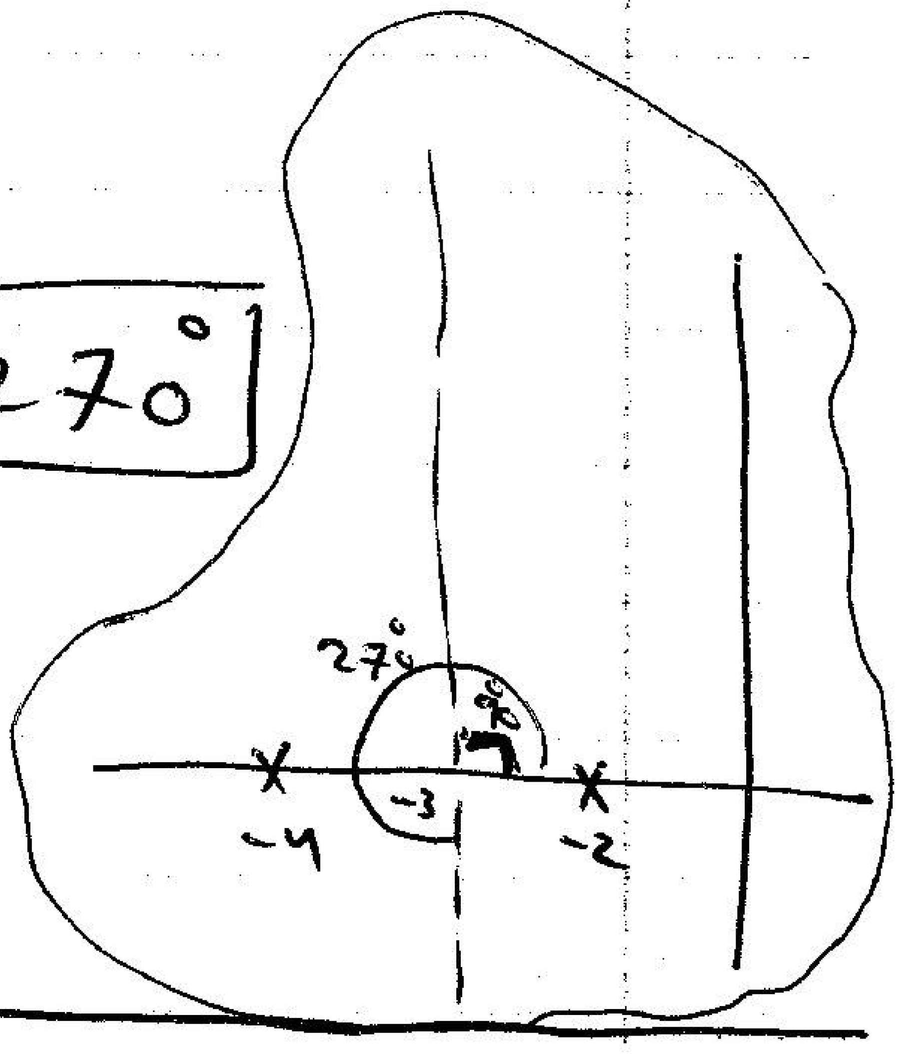
$$= \frac{-6}{2} = \boxed{-3}$$

$$\phi A = \frac{2q+1}{2} * 180 \quad ; \quad q = 0, 1$$

$$\phi A = \frac{2(0)+1}{2} * 180 = \frac{1}{2} * 180 = \boxed{90^\circ}$$

$$\phi A = \frac{2(1)+1}{2} * 180 = \frac{3}{2} * 180 = \boxed{270^\circ}$$

$$\boxed{\therefore \phi A = 90^\circ, 270^\circ}$$



$\boxed{\text{STEP 8}}$ No intersect with Imaginary - axis

$\boxed{\text{STEP 9}}$ $K = -(S+2)(S+4)$

$K = -S^2 - 6S - 8$

$\frac{dK}{dS} = -2S - 6 = 0 \Rightarrow -2S = 6 \Rightarrow \boxed{S = -3}$

صدفة أنّ نقطة ~~Breakaway~~ Breakaway هي نفسها نقطة Sentroid σ_A

اذا كان عندنا نقطتين مثلاً $S=-3$, $S=-5$

واحدة منهن ستكون Breakaway ،

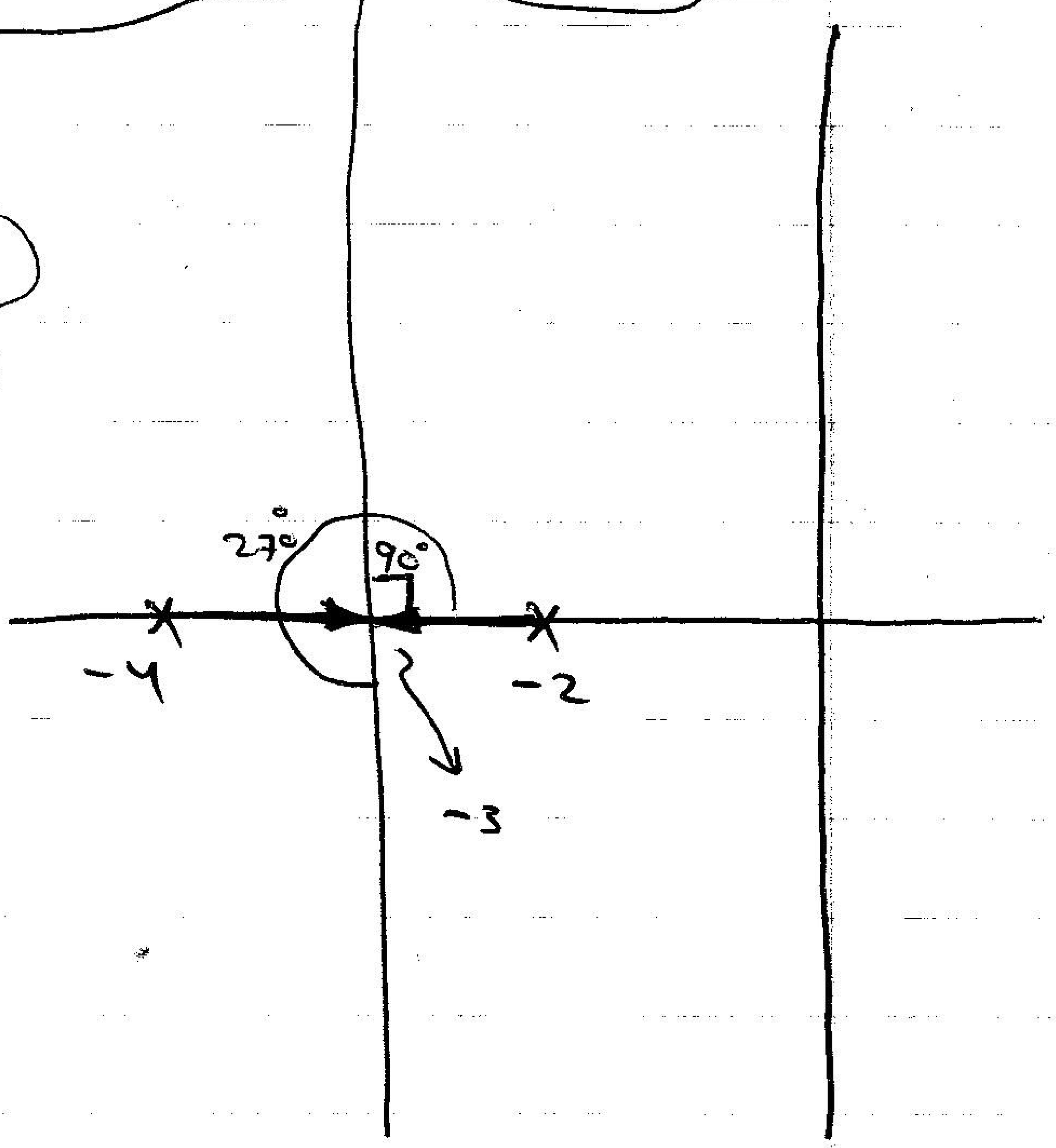
والاخرى ستكون Break in

كيف نعرف ذلك ؟

نعوض قيمة S في الاقتران

قيمة K الاكبر هي Breakaway

قيمة K الاصغر هي Break in



<u>Ex</u> find the root Locus of the following systems

$$1 + K \frac{(S+1)}{S(S+2)(S+4)^2} = 0$$

K	0	0.412	0.42	0.417	0.39
S	-2	-2.4	-2.45	-2.5	-2.1

$\boxed{\text{STEP 1}}$ already done

$\boxed{\text{STEP 2}}$ already done

$\boxed{\text{STEP 3}}$ Poles: $S = 0, -2, -4, -4$

Zeros: $S = -1$

$\boxed{\text{STEP 4}}$ locating segments on real-axis

$\boxed{\text{STEP 5}}$:- No of Separate Loci = No of poles = $\underline{4}$

$\boxed{\text{STEP 6}}$ To be Taken in Consideration

$\boxed{\text{STEP 7}}$ $\sigma_A = \dfrac{\Sigma \text{Poles} - \Sigma \text{Zeros}}{N_P - N_Z} = \dfrac{0-2-4-4-(-1)}{3} = \boxed{-3}$

$\phi_A = \dfrac{2q+1}{N_P - N_Z} * 180 \quad , \quad q = 0, 1, 2$

$\phi_A = \dfrac{2q+1}{3} * 180 \Rightarrow \phi_A = \underline{60}, \underline{180}, \underline{300}$

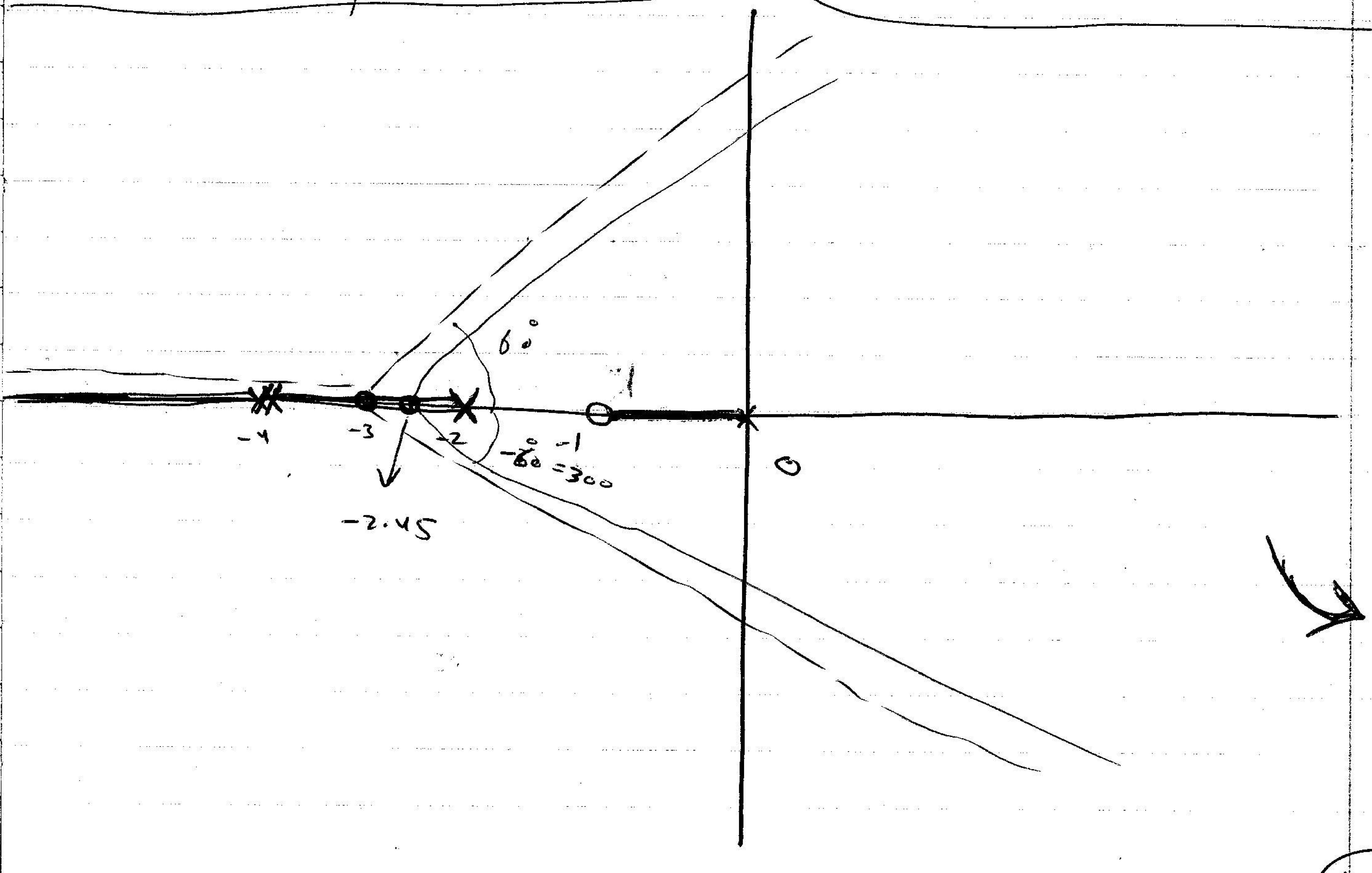
$\boxed{\text{STEP 8}}$ $1 + \dfrac{k(S+1)}{S(S+2)(S+4)^2} = 0 \Rightarrow S(S+2)(S+4)^2 + k(S+1) = 0$

at Home

لايجاد تقاطع مع محور Im

s^4	
s^3	
s^2	
s^1	$1-k$ 0
1	

take this equation and find the intersection Points with Imaginary-axis



Break away point is

$\boxed{\text{STEP 9}}$ $S = -2.45$

Back To General STEPS

$\boxed{\text{STEP 10}}$

Determine the angle of departure & ~~the~~ the angle of arrival Using the phase angle criteria .

انطلاق / الوصول

$$\phi = \pm 180 \,(2q+1)$$

take $\phi = \pm 180$ at $q = 0$
$\boxed{180}$

$\boxed{\text{Example}}$ $GH(s) = \dfrac{K}{(S+P_3)(S^2 + 2\zeta w_n S + w_n^2)}$

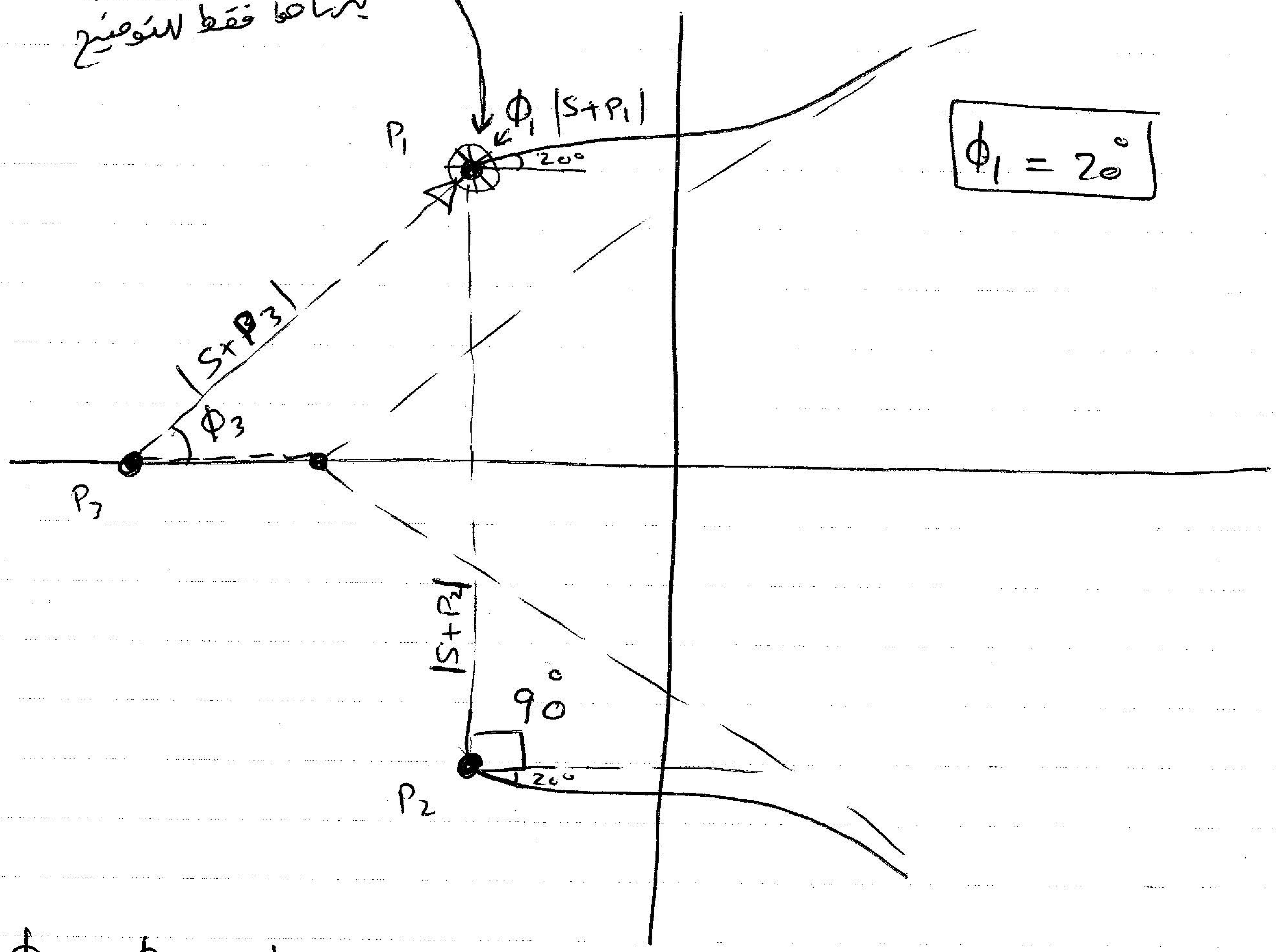
$$= \frac{K}{(S+P_3)\underbrace{(S+P_1)\;(S+P_2)}_{\text{Pole Conjugate}}}$$

$$\sigma A = \frac{P_1 + P_2 + P_3}{3-0}$$

$$\phi_A = \frac{(2q+1)}{3} * 180 \qquad (q = 0, 1, 2)$$

$$\boxed{\phi = 60, 180, 300}$$

هي عبارة عن دائرة نصف قطرها صفر
نرسمها فقط للتوضيح



$\boxed{\phi_1 = 20°}$

$$\phi_1 + \phi_2 + \phi_3 = 180$$

$$\phi_1 + 90 + \phi_3 = 180$$

$$\phi_1 = 90 - \phi_3$$

Suppose that $\boxed{\phi_3 = 70°}$ مجرد افتراض

$$\boxed{\therefore \ \phi_1 = 90 - 70 = 20°}$$

angle of departure : زاوية المغادرة
for poles
X⟶

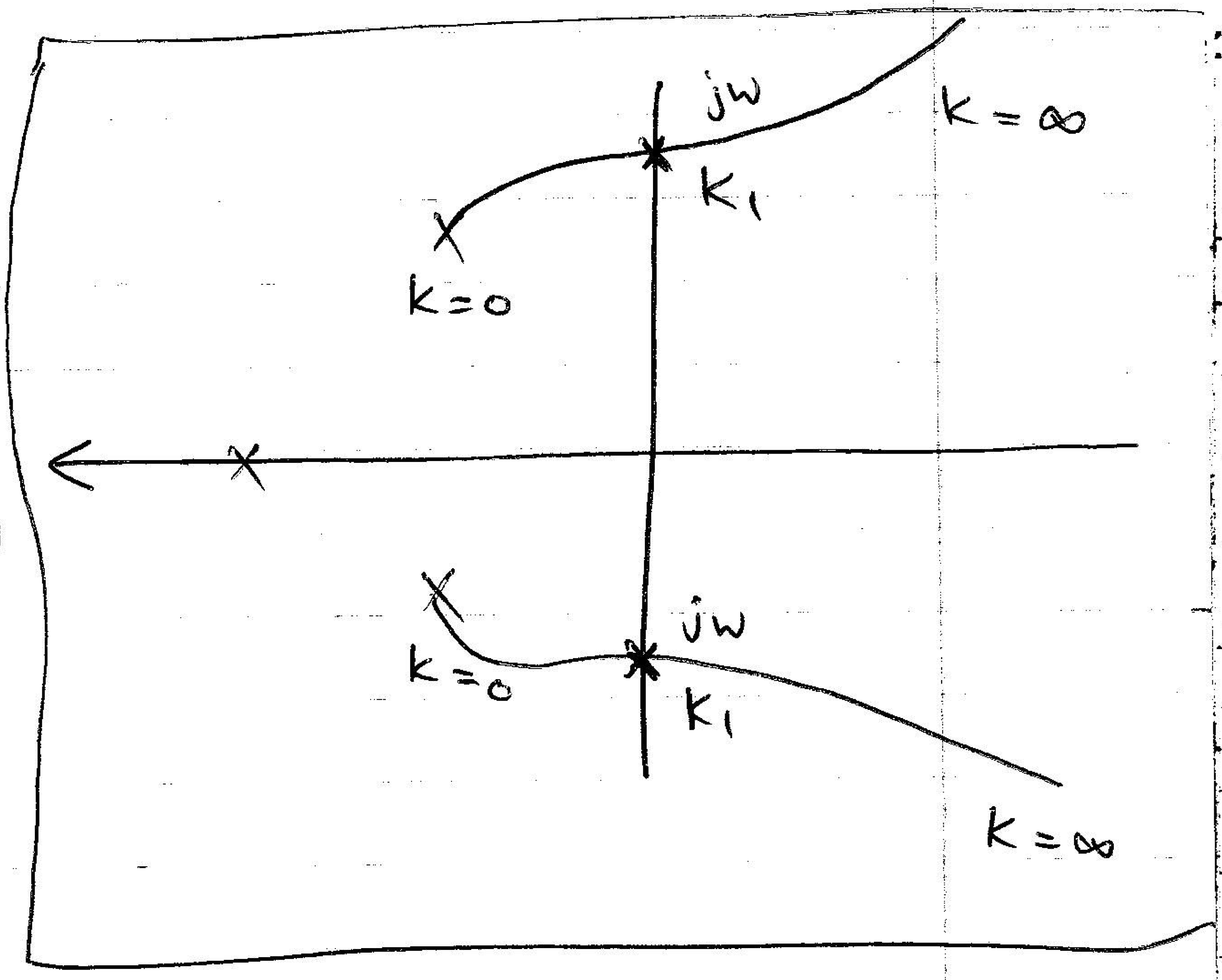
angle of arrival : زاوية الوصول
for zeros
⟶O

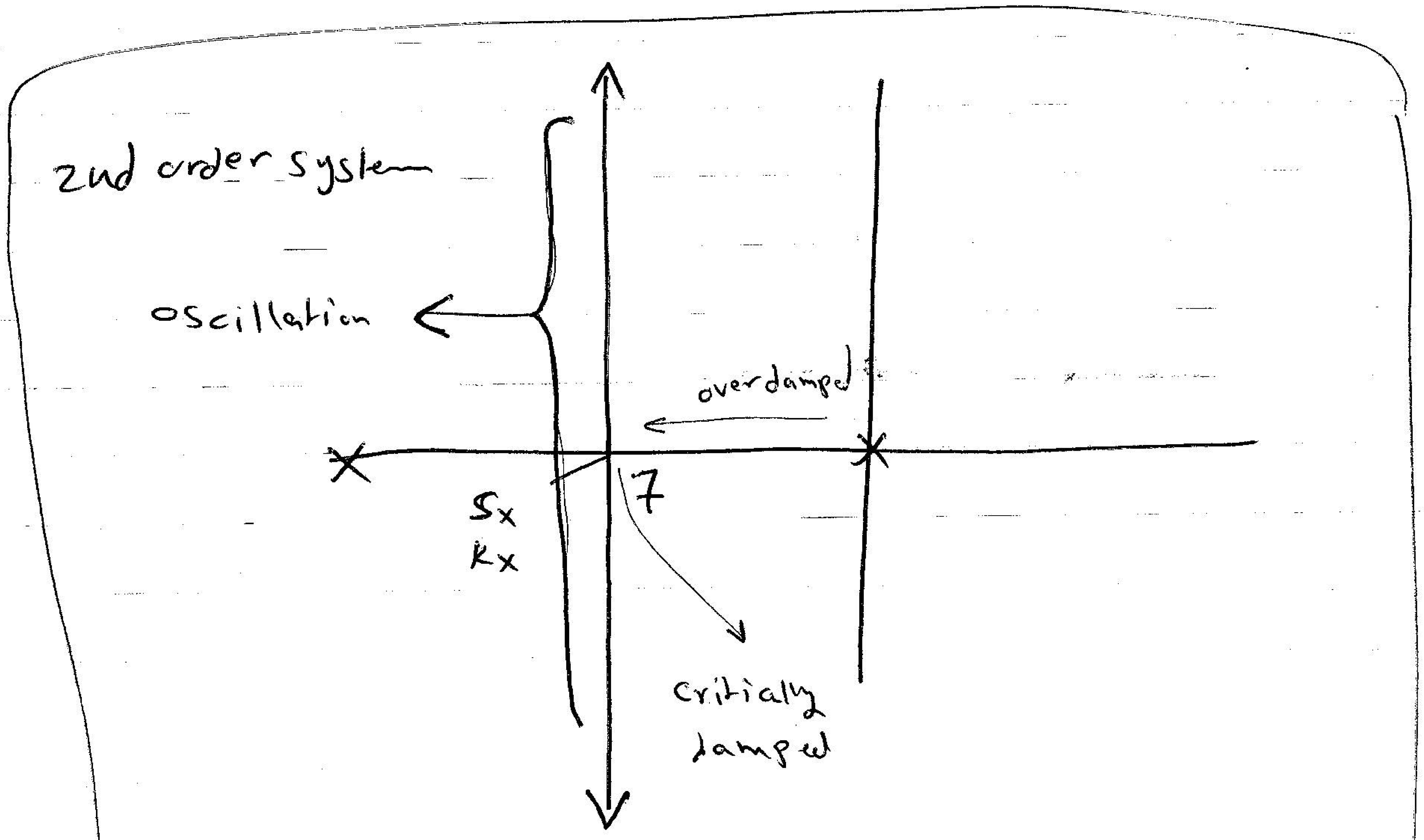
STEP 11 PLOT (take in Consideration all the previous steps and plot the system).

STEP 12

Determine the parameter Value K_x & specific root S_x

$$K = \frac{\prod_{j=1}^{n} (S + P_j)}{\prod_{i=1}^{m} (S + Z_i)}$$





$\boxed{\text{Example}}$ plot the root locus for the ch. equ., as K varies from zero to infinity

$$q(s) = 1 + \frac{K}{s^4 + 12s^3 + 64s^2 + 128s}$$

$\boxed{\text{STEP 1}}$ DONE

$\boxed{\text{STEP 2}}$ DONE

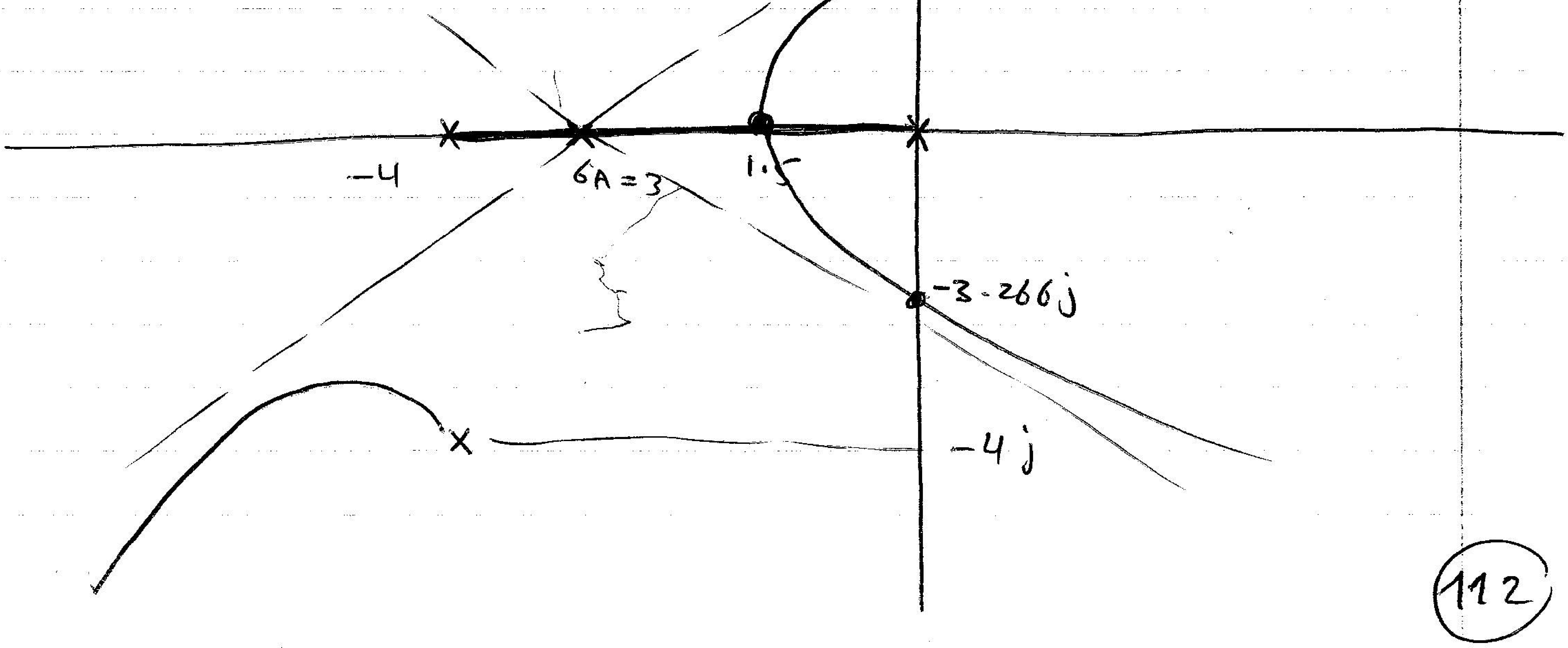
$\boxed{\text{STEP 3}}$ ~~No Zeros~~

~~Poles~~ $1 + \dfrac{K}{s(s+4)(s+4+4j)(s+4-4j)}$

$\boxed{\text{STEP 4}}$ Zeros : No Zeros

Poles : $s = 0, -4, -4-4j, -4+4j$

$\boxed{-4 < \text{Segment} < 0}$



STEP 5 Number of Separate Loci = N_p = 4

STEP 6 : Tobe taken in consideration.

STEP 7 $\sigma_A = \dfrac{\sum poles - \sum zeros}{N_p - N_z} = \dfrac{-4 + -4 + -4 + 0}{4} = \boxed{-3}$

$\phi_A = \dfrac{2q+1}{4-0}$ $(q = 0, 1, 2, 3)$

$\boxed{\phi = 45^\circ, 135^\circ, 225^\circ, 315^\circ}$

q = 0 1 2 3

STEP 8 ch. equ. $q(s) = s^4 + 12s^3 + 64s^2 + 128s + k = 0$

s^4	1	64	k
s^3	12	128	0
→ s^2	53.33	k	0
s^1	C_1	0	
1	k		

دائما تكون المعادلة التربيعية Auxiliary equation

C_1 must be zero ⇒ k = 568.89

استقرار هنا نقطة
0 → k

∴ Auxiliary Equation :-

$53.33 s^2 + 568.89 = 0 \Rightarrow s_{1,2} = \pm j\, 3.266$

STEP 9

K	0	75	85	80	68	51	0
S	0	-1	-1.5	-2	-2.5	-3	-4

$\boxed{\text{STEP 10}}$ Angle of Departure $\Phi = \boxed{225^\circ}$

$\boxed{\text{STEP 11}}$ Plot (go back to the plot page 112)

$\boxed{\text{STEP 12}}$ ~~For~~

$\boxed{\text{Stable for } 0 < K < 568.89}$

إلى هنا امتحان السكند

بالتوفيق للجميع

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